

An Approach to Modeling the Cost-Strength-Weight Tradeoff in Aluminum and Magnesium Extrusions for Automotive Applications

by

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Johann Kasper Komander

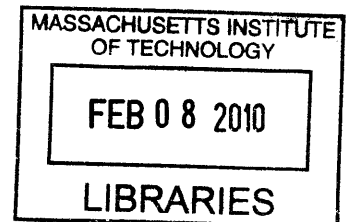
Submitted to the Department of Materials Science and Engineering in Partial Fulfillment
of the Requirements for the Degree of

Bachelor of Science

at the

Massachusetts Institute of Technology

June 2009



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Abstract

In light of volatile fuel prices and tightening emissions regulations, automobile manufacturers have been increasingly considering the use of light-weight magnesium in their efforts to improve fuel economy. While mainly used in minor components now, greater weight savings lie in its replacement of heavier structural components now made of extruded aluminum and stamped steel. However, as a material with generally lower mechanical properties on a volumetric basis and higher unit materials cost, magnesium introduces a strength-weight tradeoff with non-obvious total cost implications. Accordingly, manufacturers could greatly benefit from a method of systematically studying this weight-strength relationship in cost terms for extruded magnesium beams in a variety of loading scenarios. In this paper, we describe the development of an interface within a Process Based Cost Model of the extrusion process for quantifying these relationships on user defined parts. This interface consists of Visual Basic functions which dynamically compute dimensions of hollow Mg or Al extruded tubes necessary to achieve some strength constraint, input them into the cost model, and return the results. This capability was demonstrated on a representative system - a 1 m long, 70 or 75 mm wide, 6 or 8 mm thick Mg or Al tube – for three distinct loading conditions - axial loading as quantified by Euler buckling load, deflection from center load, and deflection from end load. Results show that in non-package constrained scenarios, cost and weight savings can be achieved by switching from Al to a larger diameter Mg extrusion of equivalent strength; however, when diameter is constrained, it is neither cost nor weight-effective unless some geometric, processing, or strength constraint is somewhat relaxed. In general, switching to Mg is favorable when *specific strength* rather than *absolute strength* is more important. While intrinsic characteristics of the model limit practical usefulness in some cases, it is nevertheless very helpful in studying *relative* differences between the strength, weight, and cost of extruded Mg and Al beams.

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Contents

LIST OF FIGURES.....	4
LIST OF TABLES.....	6
ACKNOWLEDGEMENTS.....	7
1 INTRODUCTION.....	8
1.1 Motivation.....	8
1.2 Problem Statement and Goals.....	12
2 THEORY AND METHODOLOGY.....	13
2.1 Description of Extrusion.....	13
2.2 Process Based Cost Modeling and the MSL Extrusion Model.....	17
2.2.1 Operational Principles.....	17
2.2.2 Model Layout and Tab Functionality.....	21
2.2.3 Example Calculation Flow.....	23
2.3 Structural mechanics of three distinct loading conditions.....	25
2.3.1 Axial Loading.....	25
2.3.2 Center Loading.....	27
2.3.3 End Loading.....	28
3 INTEGRATED COST AND STRUCTURAL ANALYSIS MODEL.....	29
3.1 Interface.....	29
3.2 Mechanics Functions.....	30
3.3 Integrated Cost-Structural Functions.....	33
3.4 Cost Calculations Subroutines.....	34
3.5 Press Size Determination.....	36
4 CASE STUDY.....	38
4.1 System and Assumptions.....	38
4.2 Sensitivity to Input Parameters.....	41
4.3 Axial Loading and Cost Implications of Euler Buckling Load Constraints....	44
4.4 Constant Center Loading with Variable Deflection.....	49
4.5 Constant Deflection under Varying Center Loads.....	52
4.6 Constant End Loading with Variable Deflection.....	54
4.7 Constant Deflection under Varying End Loads.....	56
4.8 Case Study Summary.....	57
5 DISCUSSION.....	58
6 CONCLUSIONS AND FUTURE WORK.....	61
7 REFERENCES.....	62
8 APPENDIX.....	63

List of Figures

Figure 1	General view of a 9-MN (1000-ton) hydraulic-extrusion press [9]	10
Figure 2	Direct Extrusion [9]. In direct extrusion, a ram forces a metal billet under high pressure through a die of desired cross section.....	13
Figure 3	Ram pressure as a function of ram stroke [9]. Ram pressure initially rises, falls, and then rises again as frictional forces change.....	14
Figure 4	Mandrel used to extrude hollow sections [8]. A simple modification to the dummy block allows for the extrusion of hollow parts.....	14
Figure 5	Extrusion constant k for various metals at different temperature [9]. The extrusion constants at 400°C for Al and Mg are ~ 68.9 and 13.8 MN/m ² respectively.....	17
Figure 6	Simplified Extrusion Cost Model Structure. Material, part, and processing parameters define various intermediate quantities which drive different types of cost.....	19
Figure 7	Example distribution of costs by type and by subprocess. The MSL Extrusion cost model separates cost by type, such as material and labor, and process, such as preheating and extrusion.....	20
Figure 8	Economies of scale in production of 70mm diameter, 10 mm thick, 1 m long Mg tube. Cost per unit decreases exponentially as production volume increases.....	20
Figure 9	Organization of “Model” tab. The cost model breaks cost down into each of seven processes moving left to right with associated parameters on the far left.....	21
Figure 10	Screenshot of extrusion data table. This table contains all necessary parameters associated with different sized presses, such as billet dimensions, weight, and cost.....	22
Figure 11	Simply supported column subjected to axial load F [10]. A column of sufficiently high slenderness ratio will buckle at the critical Euler load.....	26
Figure 12	Doubly end-supported beam under center load [12]. Maximum deflection under center load is derived by twice integrating the moment.....	27
Figure 13	Cantilever deflection by end load F [13]. A beam is weaker in end loading and hence deflects more than in center loading.....	27
Figure 14	Screenshot of <i>Dyanmic Beam</i> Table. The ‘Strength Analysis’ tab interacts with the cost model exclusively through the values in this table.....	30
Figure 15	<i>Cost Calculations</i> and <i>Deflection Cost Calculations</i> input tables. These tables specify all parameters for the cost subroutine macros.....	35
Figure 16	Drop-down menu for method of determining press size. Press size can be determined by one of three ways – via <i>PressSize</i> , <i>CCD & Wall Thickness</i> , or by manual override.....	37
Figure 17	Extrusion Rate Sensitivity. All else constant, cost decreases with increasing extrusion rate.....	42
Figure 18	Cost Sensitivity to Mg Billet and Scrap Price. Cost is highly dependent on material cost and hence billet prices and weakly dependent on scrap price.....	43
Figure 19	Mass Sensitivity to Thickness and Diameter. Volume and hence mass increases much faster with increasing thickness than increasing diameter.....	44
Figure 20	Comparison of Cost to Axial Strength – Constrained Diameter. Mg costs more than Al at constant diameter, although both become cheaper as diameter is increased....	45
Figure 21	Cost vs. Weight Tradeoff for Switch to Mg under Axial Loading - Constrained Diameter. Under constrained diameter, Mg costs and weights more than Al.....	45

Figure 22 Cost vs. Weight Tradeoff with 1 MN Axial Strength Sacrifice. By sacrificing enough strength, Mg becomes both cheaper and lower in weight than Al.....	47
Figure 23 Comparison of Cost to Axial Strength - Constrained Thickness. Mg offers a comparable price and lesser weight than Al at constant thickness, and both go down in cost at higher thickness.....	48
Figure 24 Cost vs. Deflection under 20 kN Center Load – Constrained Diameter. Cost decreases with increasing allowed deflection, with Mg being more costly than Al.....	50
Figure 25 Cost vs. Weight Tradeoff for 20 kN Center Load – Constrained Diameter. Mg weighs more than Al but this effect is lessened at higher thicknesses.....	50
Figure 26 Comparison of Cost to Deflection under 20 kN Center Load – Constrained Thickness. Mg is comparable in cost and weighs less than Al for constrained thickness under center loading.....	51
Figure 27 Cost vs. Center Load Associated with 5% Deflection – Constrained Diameter. Al is favorable to Mg at the same diameter, but if Mg is allowed to be slightly larger, it becomes cost and weight effective.....	52
Figure 28 Cost vs. Center Load Associated with 5% Deflection – Constrained Thickness. Mg weighs less and has similar cost at same thickness, but can be made more or less cost effective by altering extrusion rate.....	53
Figure 29 Cost vs. End Deflection - Constrained 70 mm Diameter. Mg is unfavorable at constrained diameter, but can become more desirable if strength constraint is relaxed....	54
Figure 30 Cost vs. Deflection Associated with 10 kN End Load – Constrained 6mm Thickness. Under constant end load, Mg is favorable to Al at constrained thickness.....	55
Figure 31 Cost vs. End Load – Constrained 70 mm Diameter. While Mg costs and weighs more at 70 mm, a weight loss is obtained by a slight strength sacrifice.....	56

List of Tables

Table 1 Properties of Magnesium Attractive for Automotive Applications [1].....	9
Table 2 Comparison of Properties between Aluminum and Magnesium. While weighing less than Al, Mg has lower modulus and yield strength.....	11
Table 3 Elements of manufacturing considered in MSL Extrusion Cost Model. The MSL Extrusion cost model breaks cost into eight fixed costs and four variable costs.....	18
Table 4 Key material dependent assumptions.....	39
Table 5 Key material independent assumptions.....	40

Acknowledgements

I would like to express my gratitude to Professor Randy Kirchain for giving me the opportunity to work on this fascinating and engaging thesis project. It allowed me to combine my interests in theoretical modeling and practical application within a useful economic context. Additionally, I owe much thanks to Sup Akamphon for his guidance which proved to be particularly helpful in giving me direction earlier in the semester. I also want to thank Richard Roth for asking all the insightful questions in our weekly meetings which ultimately motivated the case study and gave this thesis practical significance. I'm also thankful for his comments during the writing and revision of this paper. I also owe Trisha Montalbo gratitude for helping me learn the mechanics of the cost model, as well as the rest of the members of the Materials System Laboratory's cost modeling subgroup for their helpful comments and suggestions. I also thank Scott Burkett of Ube Machinery America for graciously providing me with extrusion press data and giving some practical advice. Finally, I want to thank my family for their loving support throughout my life, and my friends for making the last four years at MIT an experience I will never forget.

1 Introduction

1.1 Motivation

As the lightest of all structural metals, magnesium has emerged in recent years as a promising strategy for reducing the weight of automobiles. In light of recently volatile fuel prices and increased government regulations of greenhouse gas emissions, vehicle light weighting is increasingly being seen as an effective strategy to improving fuel efficiency [1,2,3,4]. European automakers for example have collectively promised to reduce CO₂ emissions from 170 g/km down to 120 g/km by 2012. Under the accepted assumption of “100 kg less weight of car body reducing fleet fuel consumption by 0.3-0.5 l/100 km,” this will require a decrease in total car weight of ~30% [2].

In the US, there have been steady efforts in this regard beginning with the Energy Policy and Conservation Act of 1975 which established nationwide standards for automotive fuel efficiency, known widely by their acronym CAFE [1]. Since then, standards have been raised several times, and corporations have invested heavily in R&D to avoid penalties for non-compliance. However, a combination of frozen standards and lower fuel prices lead to stagnation in fuel efficiency gains in the 1990s [1]. It has been only in the last few years, in light of a weakening American auto industry, foreign innovation, and greater socio-political fear of global warming, that fuel efficiency gains have reemerged as a top priority among American automakers. In their view, light weighting is a promising first step in moving toward the new generation of ‘green’ cars.

In industry, light weighting is largely thought of as a means to achieve performance improvements, not limited to, but including fuel economy. A general rule of thumb, the 10-5 rule, states that a 10% reduction in mass will result in a 5% increase in fuel economy [1]. To achieve these higher performing, more fuel-efficient designs, American auto makers have increased their use of light-weighting materials, as evidenced by a 50% increase in the use of polymer composites and a 150% increase in the use of aluminum since 1977 [1]. However, these statistics primarily reflect increased use in non-structural applications. Much greater weight savings can be achieved by switching to magnesium

and its alloys for the heavier components of vehicle body structure and chassis. While there has been some penetration of these technologies into mainstream vehicle body structures, the Audi Space Frame ASF being a prime example, the vast majority of vehicle production has made little use of non-ferrous lightweight structural materials in the body structure, especially in the US [2]. Further, magnesium's exceptional specific strength and manufacturing properties can translate into significant mass and manufacturing cost savings. Several of the properties which make magnesium attractive in automotive applications are provided in Table 1.

Table 1: Properties of Magnesium Attractive for Automotive Applications [1]

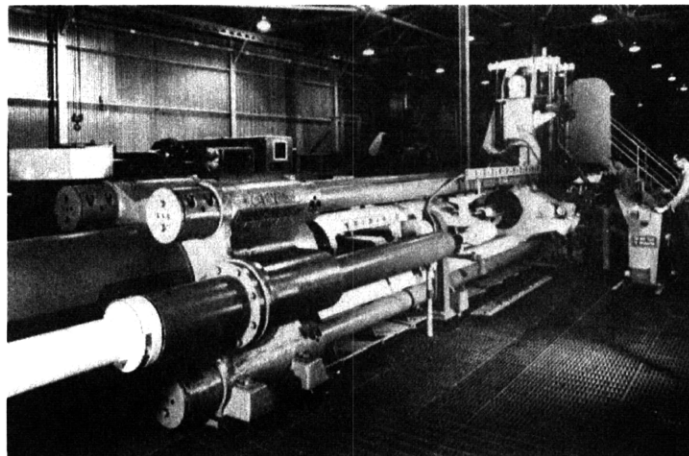
<u>Property</u>	<u>Engineering Benefit</u>
Low Density	2/3 that of Al and 1/4 of steel, enables weight reduction and improved fuel economy
High Ductility when Heated	Higher than Al, thus lower extrusion pressures needed which means longer tool life
High Specific Strength	Highest strength/weight of structural metals, esp. as alloy
Part Consolidation	Like Al, can replace multiple steel stampings with single casting, or complex shapes can be formed through bendings
Good Damping Properties	Part consolidation enabled by Mg (or Al) lead to superior NVH in body, steering, and suspension to steel

Despite these properties, magnesium has not been widely used in recent decades due to cost, technical, and compatibility limitations, and as a result, the supply base is small and non-competitive, itself leading to higher price volatility [1]. As a result, the goals of weight reduction and cost reduction have typically run counter to each other. While magnesium is already widely used in small casting applications such as instrument panels, steering wheels, and steering column supports, these issues have deterred auto makers away from using it more extensively in heavier body, powertrain, and chassis applications. Indeed, while magnesium usage increased eight fold between 1977 and 2000, it still comprises less than 0.3% (6 lbs) on average of the total mass of a car [5].

However, stricter fuel economy standards, higher fuel costs, and changing public attitudes have increased demand for magnesium parts in several industries. This has catalyzed technical improvements in the refining process, namely the carbothermic

'pidgeon' process, helping to replace the capital intensive electrolytic processes which have long lead times for introducing new capacity and require large capital investments [17]. Such improvements are driving the current expansion of the magnesium supply industry [1]. As a result, magnesium prices and associated volatility are likely to improve to some degree in the coming decades, thereby improving the economic feasibility of using magnesium and its alloys to replace extruded steel and aluminum parts. Examples include steering column supports, cross car structural members, body pillars, components of the engine cradle, suspension links, and even the entire body structure through the use of spaceframe concepts [1,4].

Figure 1. General view of a 9-MN (1000-ton) hydraulic-extrusion press



Increased use of magnesium in structural applications now only requires an improved materials cost situation enabled by advances in primary production, but also needs advances in forming technologies for creating final products. Casting has long been the main processing technique, accounting for 95% of magnesium use [4]. Advances in extrusion and even warm forming or stamping offer new opportunities to use magnesium. However, the introduction of these technologies not only requires technical advances, but also new, potentially large capital investments. Indeed, a standard 2000 US Ton aluminum extruder complete with foundation work, rigging equipment, and handling system can easily exceed \$4 million, and equipment capable of producing magnesium components can cost much more due to the need for more specialized equipment for

which there are often few suppliers [6]. Hence, for manufacturers to switch to Mg, the weight savings and improved manufacturability must justify the significant upfront investments and higher unit materials cost. The current economic and political climate is providing an impetus for such cost vs. performance analysis and is driving an expansion in the use of extruded magnesium for structural members in a variety of automotive applications.

The prospect of switching to magnesium extrusion introduces a whole host of tradeoffs. In addition to the cost-weight issue, there also exist important structural and strength differences as well. While having a high strength to weight ratio, magnesium, on a constant volume basis, is weaker than aluminum or steel due to lower Young's modulus and yield strength. Depending on the part geometry and loading conditions, this has implications on macroscale deflections and deformations experienced. If parts are constrained geometrically, a reasonable assumption for automotive parts, then this is an important factor to consider.

Moreover, depending on specific processing conditions, the microstructure of magnesium (grain size, defect density, etc) can vary substantially with respect to aluminum, so the performance in tension or compression under various applied loads will differ substantially [3]. However, these differences in microstructure lead to Mg's higher ductility, which translates into the manufacturing properties which allow it to be formed into shapes that consolidate many parts into a single component in the case of casting and bent easier into more complex shapes in the case of extrusion. Further, these micro and macro structural differences are well understood in literature and optimal processing conditions (extrusion temperature and speed) can be reasonably deduced. Table 2 below offers a comparison of material properties of aluminum and magnesium.

Table 2: Comparison of Properties between Aluminum and Magnesium

<u>Property</u>	<u>Aluminum AA6060</u>	<u>Magnesium AZ31F</u>
Young's Modulus E [GPa]	70	45
Tensile strength [MPa]	210	207
Density [kg/m ³]	2700	1800
Melting Temperature [°C]	660	650

1.2 Problem Statement and Goals

Clearly, these structural and cost differences introduce non-trivial optimization problems when considering using magnesium in place of aluminum. If we assume that engineers face real-life strength and safety constraints in various loading conditions, then a simple question to ask is: what dimensions are required in magnesium to achieve same structural integrity as aluminum, and at what cost? This question is a good starting point for addressing the need to systematically model this strength-cost-weight tradeoff and optimize corresponding production decisions. More specifically, answering this simple question works to address a greater desire to define the real-life applications in which magnesium extruded parts are preferable to aluminum in terms of cost and performance.

This goal can be accomplished through Process Based Cost Modeling (PBCM) of the extrusion process combined with the development of a model for performing various strength calculations. To this end, I utilized and improved upon the Material Systems Laboratory's (MSL) existing Extrusion cost model, a Microsoft Excel model which breaks down the extrusion process into all of its constitutive steps and computes the cost for extruding a user-defined part. However, in its previous form, the cost model had no capability for understanding the strength characteristics of the input parts. This therefore necessitated the development of a suite of Visual Basic functions dealing with strength calculations capable of interfacing with the cost model.

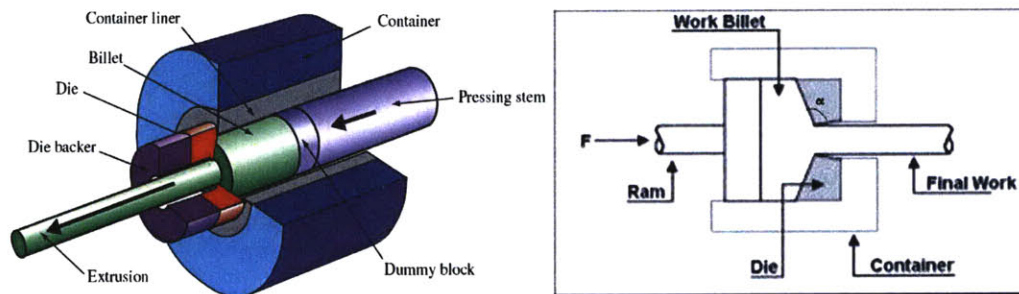
In this paper, I describe the various tools developed in order to simultaneously quantify the strength and cost of an extruded part, as defined by three common real-life loading conditions. I then demonstrate the capabilities of this method via a case study on a simple yet applicable example system. Clearly, the relevance of the resulting analysis depends highly on the quality of the inputs; however, even if absolute values are considered subjectively, this framework provides for systematic and convincing comparisons of aluminum and magnesium extrusion on a relative basis. Prior to describing this case study, an overview of the extrusion process, theory of cost modeling within the context of the MSL extrusion model, and structural mechanics relevant to three chosen loading conditions is useful.

2 Theory and Methodology

2.1 Description of Extrusion

In direct extrusion, a metal billet (usually round) is placed in a chamber and forced by a ram under high pressure through a die of desired cross section, a process analogous to squeezing toothpaste from a tube [8]. This batch or semi-continuous process is illustrated below in Figure 2. There also exist other variants of this process, namely indirect extrusion, where the die moves toward the billet, and hydrostatic extrusion, which utilizes a fluid to transfer pressure between the ram and billet. Direct extrusion is the most widely used variant however, and will thus be the focus of this discussion.

Figure 2: Direct Extrusion [9]



Advantages of direct extrusion include the variety of shapes possible, enhanced grain structure in hot and cold extrusion, and the low amount of wasted material [8]. However, a limitation is that the extruded part must have a uniform cross section throughout. Hence, this process is an ideal method of manufacturing long structural members which can be cut to desired length. With respect to automobiles, many of the solid and hollow rod-like members present in the spaceframe, suspension links, and other chassis components are commonly manufactured using direct extrusion.

Metals such as aluminum and magnesium are commonly heated to temperatures above their recrystallization temperature prior to extrusion to increase ductility and thus decrease the force required to plastically deform the metal [9]. This plastic shearing

occurs at the interface between the billet and the container wall as the applied ram pressure overcomes the material's average flow stress plus the frictional force present at the interface [8]. However, at these temperatures, oxides can form on the surface which aggravate the friction problem and introduce defects into the metal. For this reason, a dummy block of slightly smaller diameter than the billet is placed in front of the ram, so that as the metal deforms, a narrow ring of mostly oxide is left. Thus, as metal is forced is funneled into a smaller cross section, there is a lagging flow of metal at the interface approaching the back of the billet. This combined with the material left inside the die forms an unusable butt at the end of the billet, characterized by a sharp increase in ram pressure near the end of the ram stroke, as shown below in Figure 3.

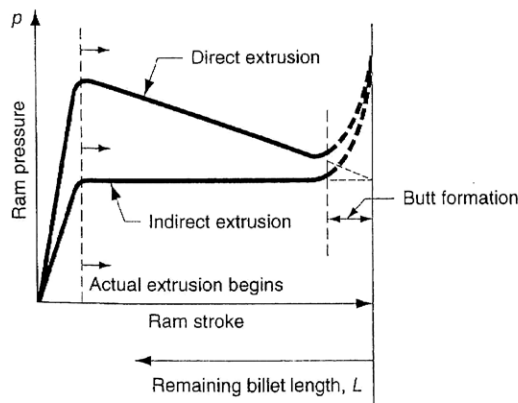


Figure 3: Ram pressure as a function of ram stroke [9]

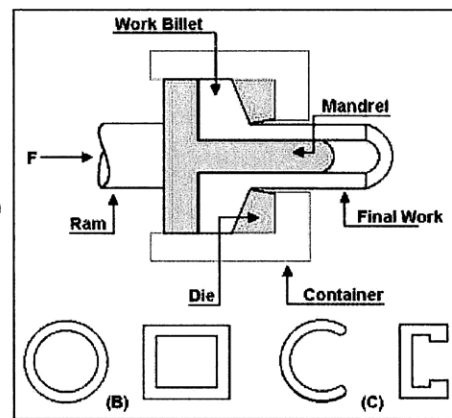


Figure 4: Mandrel used to extrude hollow sections [8]

The above figure plots ram pressure as a function of increasing ram stroke, or equivalently, decreasing remaining billet length. The initial increase in pressure is necessary to reach the material flow stress plus the initial frictional force; however, once the deformation begins, this pressure reduces due to the shrinking contact area between the billet and container walls. Hence the pressure falls until the very end of the extrusion when the butt begins forming. Hollow tubular sections are possible by attaching a mandrel to the dummy block, as shown in Figure 4 above.

The geometric variables in extrusion are the die angle, α , and the ratio of cross-sectional areas of the billet and extruded part, A_o/A_f , called the extrusion or reduction ratio R . Die angle is usually chosen by rule of thumb to optimize the tradeoff between higher surface area (and thus friction) at low die angles and the higher flow turbulence (and thus higher ram pressure) at high angles. Extrusion ratios typically range from 10-100, although values can be as high as 400 for special applications and lower for less ductile materials, although never below 4 - the minimum value necessary to deform the material plastically through the entire billet [9]. Extruded products generally range from 7.5 to 30 m in length, with shorter members being cut from longer extrusions.

The main parameter for describing the final product shape is the circumscribing-circle diameter (CCD), which is the diameter of the smallest circle which completely encloses the cross section of the part. CCDs for aluminum and magnesium typically range from 6 mm to 1 m (0.25 in. to 40 in.), although most are within 0.25 m (10 in) [9]. Furthermore, for non-solid, round extrusions, there is an increase in frictional contact area due to the additional inner perimeter of the cross section. This added force requirement is quantified by the shape factor, which is related to the ratio of the perimeter of the extruded part C_x to that of a circle of equivalent area C_c . A solid round extrusion has a minimum shape factor value of 1, while increasing complex shapes will have higher values. From literature [8], we can express the shape factor as:

$$K_x = 0.98 + 0.02 \left(\frac{C_x}{C_c} \right)^{2.25} \quad (1)$$

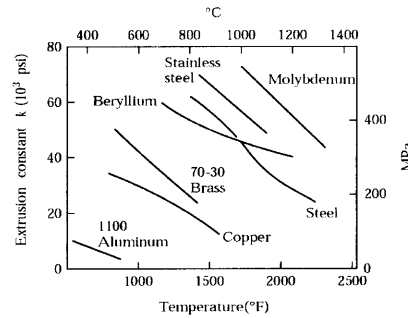
The main operating parameters are the extrusion speed and temperature. Extrusion speeds, as defined by the speed of the runout table, range up to 30 m/min (100 ft/min), generally lower for aluminum and magnesium and higher for harder steel and refractory alloys [9]. As described later on in Section 4.1, a reasonable extrusion speed for both metals was determined to be 19 m/min. Extrusion temperatures are generally chosen to be ~60-70% of the melting temperature, as the recrystallization temperature scales with melting temperature. Since aluminum and magnesium have similar melting points, they can be extruded in the same standard range of 375°C to 475°C [9].

The final parameter involved in extrusion is the actual extrusion force. The force required to perform a desired extrusion depends most directly on the reduction ratio R as well as the strength as defined by the coefficient and exponent of the material's plastic strain law $\sigma = K\varepsilon^n$, where the ideal strain ε is given by $\varepsilon = \ln R$ [8]. Assuming ideal deformation without friction, the ram pressure is thus expressed as, $p = \bar{Y}_f \ln R$, where the average flow stress is defined by $\bar{Y}_f = K\varepsilon^n / (1 + n)$. However, in reality friction between the billet and container increases the actual strain above that of the ideal value. Likewise, the pressure must overcome not only the average flow stress but also the additional friction which depends on the contact area of the billet. While analytical formulas have been developed in literature to adjust for these realities, an easier and more reliable method has been to simply encapsulate all of non-idealities into a single extrusion coefficient k obtained from measurements [9]. Some empirical values of k obtained by P. Loewenstein are provide in Figure 5 below. In this method, the extrusion pressure is defined simply as $p = k \ln R$. Utilizing the shape factor K_x to account for the additional friction arising from complex shapes, the force is thus given by multiplying the billet area by the pressure:

$$F = pA_o = A_o K_x k \ln(A_o / A_f) \quad (2)$$

While empirical data on magnesium was not found in literature, it can be estimated from the analysis above, noting that the ratio of ideal extrusion pressures for aluminum and magnesium can be approximated by the ratio of their average flow stresses. The strain laws $\sigma_{Al} = 200 \text{ MPa} \cdot \varepsilon^{0.1}$ and $\sigma_{Mg} = 40 \text{ MPa} \cdot \varepsilon^{0.09}$ obtained in literature imply a ratio of ~ 5 ; hence, the extrusion constant for magnesium can reasonably be approximated as one-fifth that of aluminum. Based on Figure 5, the extrusion constants at 400°C for Al and Mg were determined to be ~ 68.9 and 13.8 MN/m^2 respectively.

Figure 5: Extrusion constant k for various metals at different temperature [9]



Now that extrusion has been sufficiently introduced, I will turn to an overview of technical cost modeling explained within the context of the MSL Extrusion cost model.

2.2 Process Based Cost Modeling and the MSL Extrusion Model

2.2.1 Operational Principles

Process Based Cost Modeling is a modeling technique where a physical process is deconstructed into its constitutive sub processes in the effort to isolate individual cost drivers. Using industry guidelines and mathematical formulas, engineers relate part geometry and material properties to the processing parameters which ultimately define the manufacturing process and determine cost [7]. The ability to fine-tune part geometry and operational parameters allows engineers to simulate various physical manufacturing operations on computers, thus helping to avoid “time-consuming and potentially expensive prototyping” [7].

The effectiveness of these models in simulating cost lies in their recognition of the “interrelated nature of product design and production cost: while the cost of a product is a function of the process used to make it, at the same time, the cost of operating a process is a function of the design of the product being produced” [7]. In the same spirit, a PBCM feeds various user inputs, such as part material, geometry, and operating parameters, into an analytical simulation of each subprocess, which in turn computes material, energy, labor, equipment, and other relevant costs. In this way, an engineer can understand which aspects of design – whether specific part dimensions or operating conditions – drive specific as well as overall cost.

Table 3: Elements of manufacturing considered in MSL Extrusion Cost Model

Fixed Costs	Variable Costs
Main Machine Cost	Primary Material Cost
Auxiliary Equipment Cost	Secondary Material Cost
Installation Cost	Energy Cost
Building Cost	Labor Cost
Tooling Cost	
Overhead Labor Cost	
Maintenance Cost	
Cost of Working Capital	

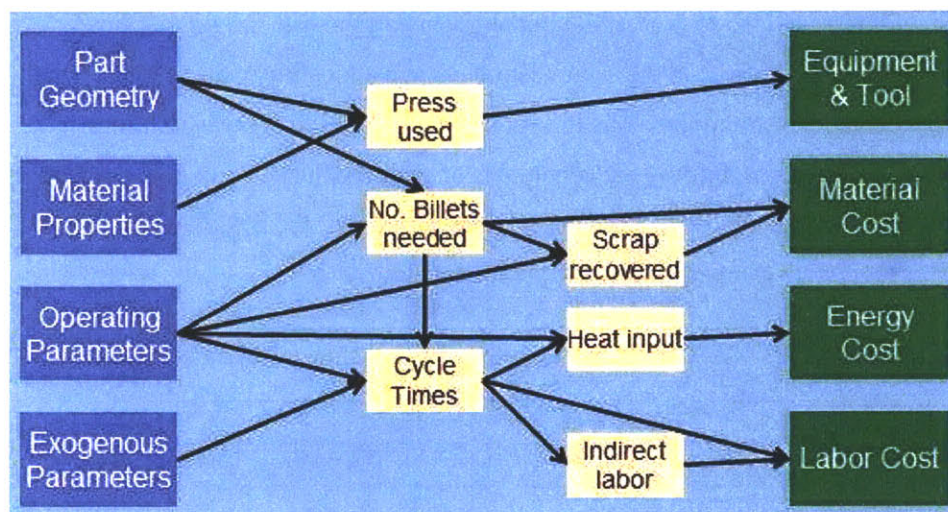
This framework is easily explained in the context of the MSL's Extrusion Model.

This model splits extrusion into seven distinct component processes – billet preparation, billet pre-heating, extrusion, run-out, bending and coating, aging, and inspection. With each of these sub processes, the cost model utilizes analytical formulas together with user defined part and operating parameters to compute twelve distinct variable and fixed costs, shown in Table 3 above, on both a per year and per unit basis. Variable costs include primary material, secondary material, energy, and labor costs, while fixed costs include the main machine, auxiliary equipment, installation, building, tooling, overhead labor, and maintenance costs, as well as the cost of working capital.

Each of the seven processes has an associated set of specific operating parameters, such as scrap and reject rates, temperature, power requirement, heat loss factor, equipment dimensions and speeds, workers required, and other parameters. Some of these, like scrap rates and temperature, are user-defined inputs governed by industry rules of thumb or empirical data, while others, such as cycle times and workers required, are determined internally based on the part geometry, material, and values of other related parameters. For example, the number of workers required depends on the number of workers per shift (a user input) and the annual required production time, itself a formula which depends on extrusion cycle time and ultimately part geometry. There also exists a global set of user-defined exogenous parameters which apply to the entire model, such as annual production volume, electricity cost, interest rate, cost of floor space, the ratio of indirect to direct workers, and working capital period, among others.

The particular set of parameters for any specific stage depends entirely on the type of physical operation which is occurring (heat application, force application, etc) and the types of corresponding costs which arise from that process. Together with user-defined part and material parameters, often supplied by an outside database, these values feed into various intermediate calculations such as cycle time per stage, indirect and direct labor requirement, scrap recovered, part weight before and after each sub process, annual production time required, fraction of line allocated, energy consumed, heat loss, and floor space allocated, among other data. These intermediate data then recombine with various exogenous parameters within the various specific cost calculations. In general, variable costs scale linearly with some metric of output - mass, energy consumed, labor required, etc - while fixed costs are amortized over the life of capital, whether a piece of equipment, tool, or building. A simplified illustration of this flow of information is depicted in Figure 6 below. While the graphic concerns only the most important costs, in reality, this structure applies to the entire spectrum of subcosts contained in the model.

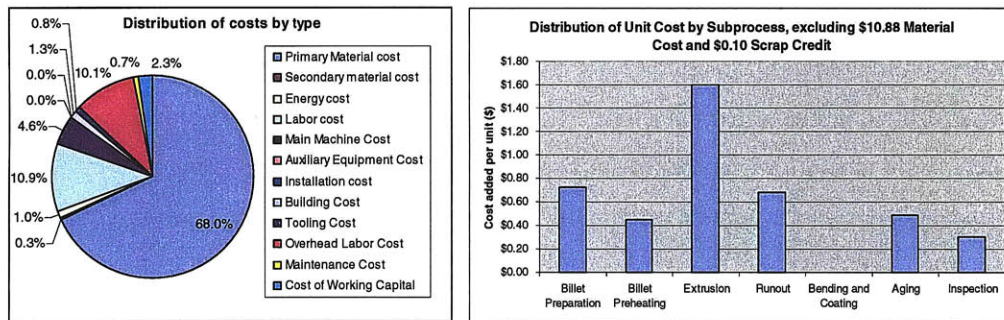
Figure 6: Simplified Extrusion Cost Model Structure



These costs are then totaled across all processes to yield a total annual and per unit cost for the extrusion specified. Furthermore, the distribution of total costs among both the twelve identified variable and fixed costs as well as among the seven distinct stages gives

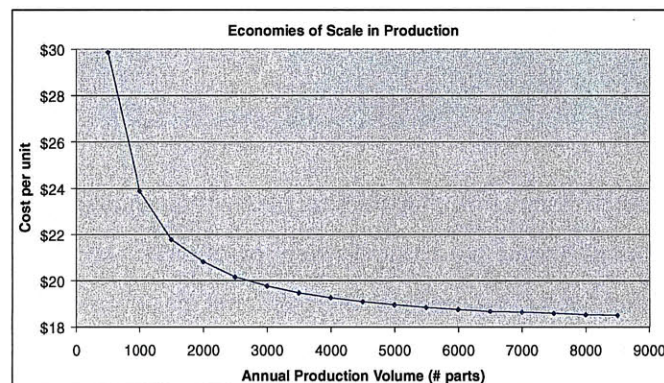
critical insight into the key drivers of overall cost. Figure 7 shows an example distribution of costs for a 50 mm diameter, 10 mm thick aluminum tube at a production rate of 40,000 per year. For the subprocess bar chart, the \$10.88 material cost and \$0.10 scrap credit were excluded since these factors exist on multiple subprocesses.

Figure 7: Example distribution of costs by type and by subprocess



An additional feature of the model is that from the annual production volume and rejection/scrap rates, one can track the quantity in kilograms as well as the number of parts entering each step of the process. Further, calculations of cycle times and the fraction of line dedicated to production of the given part provide insight into time requirements and can assist in the optimization of time allocation within a plant. This process-oriented construction results in approximating real-life cost distributions, evidenced, for example, by realistic economies of scale as illustrated in Figure 8. In general, since costs are built up from underlying parameters, the model can readily be used to explore how changes in specific parameters impact cost.

Figure 8: Economies of scale for 70 mm wide, 10 mm thick, 1 m long Mg tube

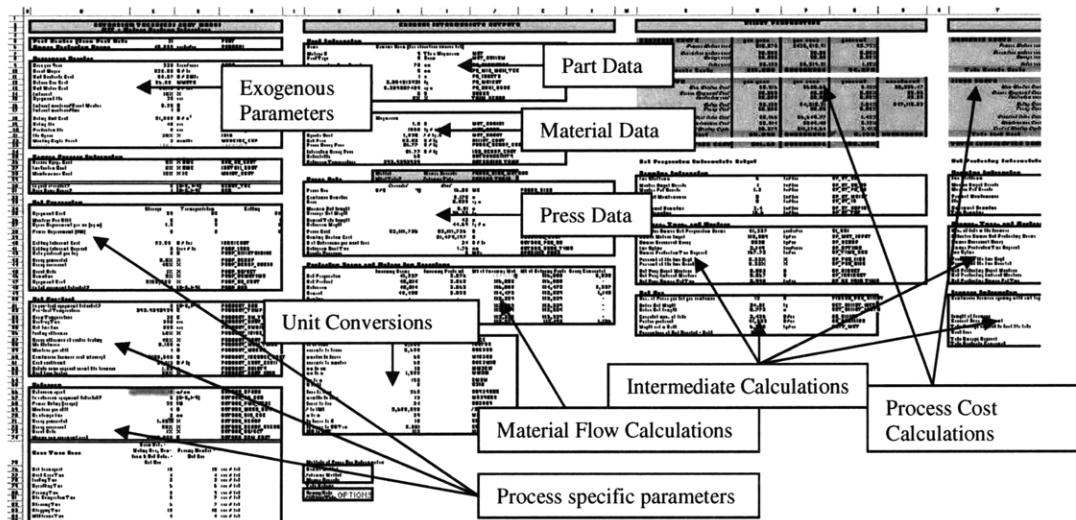


Having conceptually explained the MSL Extrusion cost model, it's now important to address some specifics regarding layout and the relationships between various tabs. This will be important for understanding the changes and additions to the model discussed in Section 3.

2.2.2 Model Layout and Tab Functionality

The MSL Extrusion cost model consists of seven tabs: “Model”, “Part Data”, “Material Data”, “Downtime”, “Extrusion Data”, “Strength Analysis”, and “Revision Notes”. The cost model itself lies in the “Model” tab, where the seven sub processes are arranged sequentially left to right with their respective intermediate calculations directly below, as shown in Figure 9. For each one, there is a corresponding table of user-defined parameters located on the left edge of the worksheet, the values of which feed into both the intermediate and cost calculations. There is also a *Part Information* table which pulls all of the data needed to define the part – notably material, CCD, wall thickness, weight, surface area, length, and cross sectional area – from the “Part Data” tab. Each row in “Part Data” corresponds to a different part while columns refer to specific geometric quantities, so depending on what part number is specified by the user, the table extracts a row of values.

Figure 9: Organization of “Model” tab



Likewise, the *Material Information* table in “Model” extracts all the material data corresponding to the part – density, specific heat, billet price, scrap price, extrusion constant, etc. – from a similarly constructed “Material Data” tab. “Downtime” feeds data into the model specifying the number of hours per day a line is idle, being maintained, shut down, etc., while “Revision Notes” is simply documentation of changes that have been made to the model. “Strength Analysis” constitutes my additions to the model for performing strength calculations and will be described in detail in Section 3.

Finally, the “Extrusion Data” tab contains two tables of parameters – one pre-existing (top) and one added as part of this work (bottom) - characterizing a number of various sized extrusion presses, as shown in Figure 10. Accordingly, the *Press Data* table in “Model” extracts the data corresponding to the specified press size, as determined by methods described later in Section 3.5. However, unlike the part and material data described above, much of these data cannot be directly computed or estimated, but rather must be obtained directly from the manufacturer. These parameters include the pressure exerted by the press, the diameter, length, and weight of the billets, the press and handling system cost, runout table length, and the number of extrusions per hour, all of which may differ between aluminum and magnesium. To acquire these data, I contacted Scott Burkett of Ube Machinery America, Inc. based in Ann Arbor, Michigan.

Figure 10: Screenshot of extrusion data table

28									
29	Press Size (UST)	UST	1804	2004	2004	2505	2756	2505	
30	Billet Dia. (in)	in	7	7	8	8	8	9	
31	Container Bore (m)	m	0.185	0.185	0.21	0.21	0.21	0.236	
32	Pressure (kg/sq mm)	kg/sq mm	60.88	67.64	52.49	65.62	72.18	51.96	
33	Container Cross Section (sq m)	sq m	26880.252	26880.252	34636.06	34636.059	34636.06	43743.54	43
34	Press Size (MN)	MN	16.05	17.83	17.83	22.29	24.52	22.29	
35	Billet Dia. (m)	m	0.178	0.178	0.203	0.203	0.203	0.229	
36	Area (sq m)	sq m	0.025	0.025	0.032	0.032	0.032	0.041	
37	Max. Billet Length (m)	m	0.813	0.851	0.851	1.003	1.054	1.003	
38	Avg. Billet Weight	kg	54.501	57.049	74.513	87.822	92.287	111.149	1
39	Runout Table Length (m)	m	42	42	42	42	42	42	
40	Extrusion Weight /m (G0)	kg / m	67.037	67.037	87.559	87.559	87.559	110.817	1
41	Press Cost	\$	\$2,074,484	\$2,304,831	#####	\$2,881,156	#####	#####	###
42	Handling system cost	\$	\$1,479,197	\$1,643,445	#####	\$2,054,390	#####	#####	###
43	Nos of extrusions per work hr		34	32	32	29	27	29	
44	Extrusion Dead Time (min)	min	1.76	1.85	1.85	2.06	2.15	2.06	
45	Specific Pressure	Mpa							
46									

In addition, he provided me with a method of estimated the cost of an extrusion press and handling equipment as a function of press size. Press and handling system costs scale

linearly with tonnage at the rates of \$1150 and \$820 per US ton respectively. While specific magnesium press data fell under the realm of proprietary information, he told me that while magnesium equipment is more complicated and expensive, you can approximate the billet dimensions and forces reasonably well with the aluminum data. Due to higher complexity and smaller supply, Mg machinery is more expensive, so the user must make an assumption regarding the scaling for magnesium equipment costs.

These data were used to construct the bottom data table in “Extrusion Data” in a similar fashion to the pre-existing top table, with billet diameter increasing moving to the right. This construction results in a 1:1 correspondence between press size and all other relevant parameters.

2.2.3 Example of Calculation Flow

With a clear picture of the model in mind, let’s walk through a sample cost calculation in order to illustrate the flow of information described in the previous two sections. Consider an arbitrary aluminum part X of some geometry as specified by the row in the “Part Data” table labeled “X.” Aluminum 6061 happens to be the material labeled “1” in the “Material Data” tab. Without going further here, let’s also say that the model determines the proper press size to be the 16.05 MN press in the bottom table of “Extrusion Data.” Hence, when the ‘Part Number’ field of the “Model” tab is set to X, all of these corresponding data are automatically loaded into the worksheet in their respective locations.

In the first step, “Billet Preparation,” over 93% of cost is material cost. This cost is calculated from the material input, itself calculated from the number and weight of billets needed to achieve the production volume, in addition to billet price, scrap price, and scrap produced, another formula. The next major cost is labor, which is computed based on two intermediate calculations – the number of direct workers and the annual number of paid hours – and the exogenous wage rate. The number of direct workers itself depends on the internally determined fraction of the line required, which feeds into other major costs – main machine (loading equipment) and building costs (dependent on square

footage of area taken up by the equipment). Their full cost is amortized over 20 years, with the result being multiplied by the fraction of line required, reflecting the fact that this line is being shared among multiple products. Maintenance cost is computed simply as an exogenously determined percentage of fixed costs, while the cost of working capital is the opportunity cost of holding cash equivalent to three months worth of variable costs.

The next step, “Billet Preheating” operates in a similar way although this time, energy and labor costs dominate. Based on input parameters such specific heat, operating temperature, heat loss factor, and heating efficiency, the total energy consumed by the extrusion is computed as an intermediate calculation. This is then multiplied by the exogenous electricity cost to yield annual energy cost. Labor cost is calculated similarly as above, with the number of workers per line being multiplied by the wage rate and the fraction of time the line is used for this particular extrusion. Based on exogenous ratio of indirect to direct workers, there is also a similar overhead labor cost. All other costs – machine (furnace), building, working capital, etc – are calculated the same as before.

Next, we have “Extrusion,” which unlike the previous steps involves intermediate calculations dealing with the extrusion press, such as reduction ratio, extrusion time, extruder power use, and the number of dies required to achieve the production volume. More importantly, as the rate-limiting process, the cycle time of extrusion ultimately define not only the energy and labor costs of this step, but also the cycle times, and hence time-dependent costs, of every other step in the process. The extrusion cycle time is determined by dividing the runout table length by the extrusion speed and adding the dead time (user input). This per billet cycle time ultimately defines the fraction of line needed and, combined with the computed die changing time, the annual production time required for the entire process. All other sub costs are calculated in the same way as before with slight changes. Notably, the amortized press cost corresponds to the press located in the “Extrusion Data” tab deemed appropriate for the extrusion by methods discussed later in Section 3.5. Energy use is determined by multiplying the annual production time by the extruder power rating and unit energy cost, while material costs become a credit due to the recovery of scrap.

Having modeled the most important physical processes in the first three steps, the remaining sub processes are less analytically intense. “Runout” is quite simple and contains only a few minor differences, namely the main machine and building costs are associated specifically with the runout table, and that there is a secondary material cost associated with runout lubricant. “Bending and Coating” deals with costs associated with the use of bending or electronic coating equipment. Aside from equipment costs, bending costs derive from the number of bends while coating costs depend on the total surface area of the extruded parts. “Aging” is a simple step where the extruded part is kept in an oven for a while to improve microstructure. Energy and labor costs parallel those of “Billet Preheating,” while machine and building costs are related to the secondary oven. “Inspection” is a simple step in which the only costs are those of labor, working capital, and a minor scrap credit due to an exogenous reject rate. Finally, the final section titled “Cost Summary” adds up all of costs through all seven processes associated with producing part X. It breaks down the cost distribution by both subprocess and by subcost.

Having introduced the mechanics of extrusion and also the structure of the MSL Extrusion cost model, it is now appropriate to introduce the metrics of strength which can be used to quantify the structural integrity of extruded parts in various loading conditions. In the effort to model typical forces experienced by structural members in vehicles, we will discuss axial loading as defined by Euler buckling load, deflection under center load, and deflection under end load. Then we will have achieved an adequate overview of theory to demonstrate, through a case study, the model’s ability to compare aluminum and magnesium extrusions in terms of cost, weight, and performance.

2.3 Structural mechanics of three distinct loading conditions

2.3.1 Axial loading

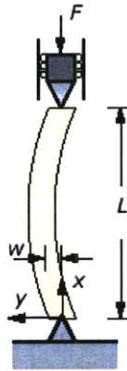
Axial strength is particularly important in applications where large loads, whether from the weight of mass or from rapid external compression such as a collision, exert large stresses along the length of an extruded member. For thin, solid and semi-hollow members, these stresses can be particularly high due to the relatively low cross-sectional area. For such stresses, the critical Euler buckling load is a useful metric for quantifying

axial strength. The Euler buckling load is the maximum compressive load a long, slender member can sustain before failing via buckling due to elastic instability, as illustrated in Figure 11 below. It is given by:

$$F_c = \frac{\pi^2 EI}{L_e^2}, \quad (3)$$

where E is the material's Young modulus, I is the area moment of inertia, and L_e is the effective length. The effective length is used to account for differences in the shape of the buckling mode due to different conditions of end support. For a hollow rod statically supported on both ends, a reasonable proxy for a structural member in a car body, the area moment and effective length are defined by $I = (\pi/4)(r_o^4 - r_i^4)$ and $L_e = L/2$ respectively.

Figure 11: Simply supported column subjected to axial load F [10]



Depending on the slenderness ratio – defined as the ratio of the effective length to minimum cross sectional radius – a column will fail via buckling, plastically deformation, or somewhere in-between. The intermediate slenderness ratio of structural columns typically means that under load a column will bend somewhat and then plastically fail. Thus, the Euler formula is not a perfect metric for axial strength, but neither is the yield stress, since, in reality, loads, such as those applied in a collision, are applied fast and without perfectly fixed end conditions. Nevertheless, it is a useful metric of comparing *relative* strengths between members. While maximizing strength might be the obvious goal for vertical members intended to hold up weight, engineers designing the chassis and frame of a car likely seek an optimal intermediate value. Ideally, a member

would be not so weak so as to fail in a fender bender, and yet weak enough to crush or bend somewhat to dampen an impact and absorb energy in a potentially fatal collision. Typical collisions involving medium sized cars traveling at 50 km/hr can range from 50 to 200 kN [11].

2.3.2 Center loading

The application of a load perpendicular to the length of a long extrusion is a common loading condition in a frame or spaceframe body structure and chassis of a vehicle, where members must endure side collisions and hold up the weight of various heavy castings such as the engine block. While in reality, such loads are distributed along the length, a reasonable first-order metric for comparing perpendicular strength of various beams is to look at the deflection that arises from a point load at the center of the beam, as illustrated in Figure 12.

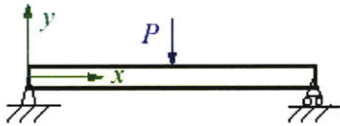


Figure 12: Doubly end-supported beam under center load [12]



Figure 13: Cantilever deflection by end load F [13]

The deflection function of an end-supported beam under center loading can be easily derived from mechanics by integrating the moment twice. Using this method, the maximum deflection is given by:

$$\delta_{\max} = \frac{FL^3}{12EI}, \quad (4)$$

where F is the applied force, L is the length, E is the modulus, and I is the area moment, as earlier described for a cylindrical beam. Likewise, by rearranging terms we can solve for applied force as a function of the max deflection, thus providing flexibility in how safety factors and strength tolerances are defined. In many applications, the structural integrity of the system depends on various beams maintaining their proper shape; hence,

the max deflection is a good metric for quantifying an extruded part's ability to withstand bending due to side forces.

2.3.3 End loading

The final loading condition to be considered in this work is that of a point load on the end of a singly-supported beam, also known as cantilever bending, as illustrated in Figure 13. End loading conditions arise in real-life during collisions where the angle of force comes from non-axial directions, in addition to structural parts which bear the weight of heavy automotive components near their end. Similar methods from mechanics applied above can be applied here as well. By equating expressions involving the curvature – the second derivative of deflection – with those involving the modulus and moments of inertia and integrating, the end deflection is found to be:

$$\delta_{\max} = \frac{FL^3}{3EI} . \quad (5)$$

This expression differs from center loading by exactly a factor of 4; hence for the same load, the maximum deflection when applied at the end will be four times that when applied in the center. In reality, end loads are somewhat distributed, thus the true behavior will lie somewhere in between these two results. Similar to above, force can be solved as a function of deflection, thus allowing engineers to calculate the forces capable of achieving various deflection tolerances.

Together these three loading conditions provide a useful analytical framework for understanding the ultimate relationship between the cost of an extruded part and its structural performance, the key link between the two being the part dimensions. While depending explicitly on length, the strength implications of various geometries arise more prominently through cross-sectional moment of inertia, which itself is a strong function of the inner and outer diameter of extruded hollow parts. In turn, the thickness strongly drives weight and ultimately the extrusion cost as reflected in material costs, force requirements, and cycle times. Combined with background theory on extrusion and cost modeling described earlier, we are now ready to introduce the technical means of understanding these relationships within the MSL Extrusion cost model.

3 Integrated Cost and Structural Analysis Model

In order for cost comparisons between aluminum and magnesium to be useful, it is critical that they be done on performance equivalent components. As such, the structural analytics described in the previous section needed to be incorporated directly into the cost model. This required that three new categories of calculations - purely mechanical functions, integrated cost-structural functions, and a press size algorithm – be integrated into the model using the functionality of Microsoft Excel.

3.1 Interface

A tab called “Strength Analysis” was created within the MSL Extrusion model for the purpose of calculating the strength characteristics of round extruded parts and dynamically interfacing with the cost model contained in the “Model” tab. Located at the top left of this tab is a table of part parameters – thickness, diameter, material, length, cross-sectional area, volume, etc. – which defines a hollow, cylindrical beam simulated by the strength functions contained in the spreadsheet, as depicted in Figure 14. This beam is called the *Dynamic Beam*. When called in the spreadsheet, these functions modify these part parameters in order to meet some user-defined performance constraint. Using Visual Basic subroutines, the “Strength Analysis” tab can dynamically feed the dimensions contained in the *Dynamic Beam* table into the “Model” tab, letting the cost model operate on that part and then retrieve the cost output.

Since part information is imported into the model from the “Part Data” tab, a special part labeled *Dynamic Beam* was added to the database in order to link the model with the “Strength Analysis” tab. Rather than being a manual input within this tab, as is the case for all other parts, the part data contained in this row references the *Dynamic Beam* table in the “Strength Analysis” tab. In this way, the “Strength Analysis” tab communicates with the “Model” exclusively through *Dynamic Beam* and only when the cost model is set to this part. This design effectively treats the cost model as a black box, thus isolating the strength analysis features and preserving the cost model’s original operation. By systematically varying the dimensions of *Dynamic Beam*, one can thus analyze and

compare the cost-weight-strength relationships for round aluminum and magnesium extruded beams.

Figure 14: Screenshot of *Dynamic Beam Table*

Dynamic Beam Table of Dimensions				Loading Analysis	
Material (# from table)		I	TEST_MAT	Applied Force	31 FORCE
Part Type (H/S)		H	TEST_TYPE	Young's Modu	70 YOUNG
Diameter (mm)		50	TEST_DIA	Area Moment	2.67E-07 MOMENT
Min Wall Thickness (mm)		10	TEST_WALL		
Length (m)		I	TEST_LEN	Cantilever End Loading	
				Max Deflection	552.81
Part Volume (m ³)	0.001257		TEST_VOL	Center Loading on Supported Beam	
Part Radius (mm)	25		TEST_RAD	Max Deflection	138.20
Part Weight (kg)	3.393		TEST_WT		
Part Surface Area (m ²)	0.251		TEST_SA	Euler Buckling	
Cross Section (m ²)	0.001		TEST_CROSS	Critical Load (t)	0.74 BUCKLING
Inner Radius	15.000		TEST_RAD_IN		
Trim Scrap Rate	5%		TEST_TRIM	Cost/unit	16.10378581
No. Bends	0		TEST_BENDS	Cost/kg	4.746290951

3.2 Mechanics Functions

With this goal in mind, a set of Visual Basic functions was developed in the MSL Extrusion cost model to compute dimensions (thickness or outer diameter) or metrics of strength (buckling load, deflection, or load required) associated with input parameters and constraints provided by the user. Some functions compute the dimensions necessary to achieve a certain metric of strength, while others compute a metric of strength given the dimensions and loading condition. These functions, described below, only perform the mechanics calculations described earlier and do not interact with the cost model at all. Please refer to the Appendix for full source code.

BucklingCalc() This function computes either the thickness or diameter (holding the other constant) needed to achieve a specific Euler Buckling strength of a given material. If the function is given a diameter (implicitly defining the outer radius), it solves equation 3 for the inner radius, as shown below, and returns the thickness $t = r_o - r_i$.

$$F_c = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E}{L/2} \cdot \frac{\pi}{4} (r_o^4 - r_i^4) \rightarrow r_i = \sqrt[4]{r_o^4 - \frac{F_c L^2}{\pi^3 E}} \quad (6)$$

Likewise, if given a thickness, *BucklingCalc* returns the outer diameter which will achieve the desired buckling strength. However, in this case the terms cannot be simply rearranged, as solving for the outer radius requires solving a cubic polynomial:

$$F_c = \frac{\pi^2 EI}{L_e^2} = \frac{\pi^2 E}{L/2} \cdot \frac{\pi}{4} [r_o^4 - (r_o - t)^4] \rightarrow 4tr_o^3 - 6t^2r_o^2 + 4t^3r_o - \left(t^4 + \frac{F_c L^2}{\pi^3 E} \right) = 0 \quad (7)$$

BucklingCalc uses the closed form solution of $ax^3 + bx^2 + cx + d = 0$ to solve this equation for r_o and then returns the diameter $D = 2r_o$. As an example, if the *Dynamic Beam* length is set to 1 m, calling the formula ‘=BucklingCalc(“Al”,”Thickness”, 85, 2.60)’ returns a value of 4.59. This is equivalent to saying that for a 1 m long hollow aluminum rod of 85 mm diameter, the thickness required to achieve a 2.60 kN Euler buckling load is 4.59 mm. Likewise, calling ‘=BucklingCalc(“Mg”,”Diameter”, 5, 2.60)’ will solve for the diameter (in mm) required to achieve a 2.60 kN Euler buckling load for a 1 m long hollow magnesium rod of thickness 5 mm. In general, the first argument is the material, (“Al” or “Mg”), the second is the dimension to vary (“Thickness” or “Diameter”), the third is the value of the fixed dimension (either thickness or diameter in mm), and the fourth argument is the desired critical Euler buckling load in kN.

DeflectionCalc() This function computes the thickness or diameter necessary to achieve a specified center or end point loading condition. It operates similarly to *BucklingCalc* but instead of having an Euler force as an input, it takes a load (in kN), loading condition (center or end), and deflection (as a percent) as inputs. It answers the questions of the form: for an Al or Mg hollow cylinder of thickness (diameter) of X mm, what is the diameter (thickness) required to achieve a Y% deflection under Z kN center or end load?

When given a constant diameter, *DeflectionCalc* solves either Equation 4 or 5 (depending on whether center or end loading is specified) for the inner radius as a function of outer radius, force, length, and deflection. For end loading for example, the calculation is:

$$\delta = \frac{FL^3}{3EI} \rightarrow I = \frac{FL^3}{3EI} \rightarrow \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{FL^3}{3EI} \rightarrow r_i = \sqrt[4]{r_o^4 - \frac{4FL^3}{3\pi E \delta}} \quad (8)$$

The procedure for solving for thickness under the constant diameter constraint is similar to that of the buckling calculations, requiring *DeflectionCalc* to solve a similar cubic polynomial. For end loading, rearranging Equation 5 yields:

$$\delta = \frac{FL^3}{3EI} \rightarrow \frac{\pi}{4} [r_o^4 - (r_o - t)^4] = \frac{FL^3}{3\pi E \delta} \rightarrow 4tr_o^3 - 6t^2r_o^2 + 4t^3r_o - \left(t^4 + \frac{4FL^3}{3\pi E \delta} \right) = 0. \quad (9)$$

Again, this can be solved using the closed form solution to the standard cubic polynomial. Both of these calculations are nearly identical for center loading, except that the 4/3 factor is replaced with a 1/3. As an example, calling the formula ‘=DeflectionCalc(“Center”,”Al”,”Thickness”, 85, 0.02, 15)’ returns the value 4.32. This says that for an aluminum hollow rod of diameter 85 mm, the thickness required to achieve a 2% deflection under a 15 kN center load is 4.32 mm. In general, the arguments are loading type (“Center” or “End”), material (“Al” or “Mg”), varied dimension (“Thickness” or “Diameter”), value of the fixed dimension (thickness or diameter in mm), deflection (expressed as a percent decimal), and applied load in kN. Since the function takes both the percent deflection and load as inputs, the user has the functionality to investigate them independently by leaving the other constant. The user can also systematically investigate the differences between holding thickness and diameter fixed, as well as the differences between center and end loading for both materials. In general, having so many independent variables provides great functionality to investigate the relationship between geometry and deflection in various loading conditions.

Deflection() This function computes the deflection percent of a specified beam under given load. For center and end loading, *Deflection* simply inserts the input values into equations 4 and 5 respectively. For example, calling ‘=Deflection(“Center”, “Al”, 1, 6, 85, 15)’ yields the result 0.0153, which simply says that the max deflection of a 1 m long, 6 mm thick, 85 mm diameter aluminum tube under 15 kN center load is 1.53%.

Load() This function is exactly the same as *Deflection* except that instead of calculating deflection as a function of load, it calculates load as a function of deflection. For

example, the formula ‘=Load("Center","Al", 1, 6, 85, 0.0153)’ gives the result 15 kN, the inverse calculation as the last example.

Weight() This function computes the weight of a hollow cylindrical rod from the density (determined by material), length, thickness, and diameter. The formula ‘=Weight("Al", 4.6, 85, 1)’ will return the weight of a 1 m long, 4.6 mm thick, 85 mm diameter aluminum tube. These five functions - *BucklingCalc*, *DeflectionCalc*, *Deflection*, *Load*, *Weight* - constitute the purely mechanics calculations in the spreadsheet.

3.3 Integrated Cost-Structural Functions

Another set of functions was developed specifically for interfacing with the cost model. These functions work by calling *BucklingCalc* and *DeflectionCalc* internally subject to specified geometric and loading constraints, inputting their results into the Dynamic Beam table, and retrieving the associated cost output.

CostCalcBuckling() This function is an extension of *BucklingCalc* which interfaces with the cost model. It works by taking the same inputs as *BucklingCalc*, feeding these inputs to it internally, pasting the output dimensions in part data for the *Dynamic Beam*, and returning the corresponding cost from the cost model, either in units of \$/part or \$/kg. Since this function modifies values contained in cells, it cannot be called as a formula in the spreadsheet; instead, it must be embedded into a Visual Basic subroutine, as described below. As an example, calling ‘=CostCalcBuckling("Mg", "Thickness", 85, 2.7, "unit")’ within a Subroutine will give the result 33.53, which says that for a 85 mm diameter magnesium tube with thickness such that the Euler buckling load is 2.7 MN, the per unit cost is \$33.53. Likewise, *CostCalcBuckling* can also be used to determine the cost of extrusions where the thickness is fixed and diameter allowed to vary in order to meet the strength constraint.

CostCalcDeflection() This is identical to the previous function except that it internally calls *DeflectionCalc* instead of *BucklingCalc*. Depending on the inputs provided, it will thus implicitly determine the thickness or diameter (depending on which is held fixed)

necessary to achieve a center or end loading constraint (as defined by a deflection percent and kN load) for either Mg or Al. It then pastes these values into the *Dynamic Beam* table in the “Strength Analysis” tab, lets the cost model recalculate, and, depending on the type of cost specified, returns either the unit cost or cost per kg for the given extrusion. Again, it must be called within a subroutine. Revisiting the example discussed earlier for *DeflectionCalc*, calling ‘=DeflectionCalc(“Center”,”Al”,”Thickness”, 85, 0.02, 15, “unit”)’ returns a per unit cost of \$23.41, but implicit in this calculation is the determination by *DeflectionCalc* that the appropriate thickness to achieve the strength constraint is 4.32mm. Together these two functions are solely responsible for feeding in and retrieving data from the cost model.

3.4 Cost Calculation Subroutines

As described above, Visual Basic subroutines were developed in order to perform a user-defined sequence of cost calculation operations. Subroutines exist in Visual Basic as a key, thread-protecting element of code structure, ensuring that parallel lines of code are not modifying the same cell in the spreadsheet simultaneously. This is necessary in order to ensure that all references in the spreadsheet are uniquely defined at any time. If, for example, the *CostCalc* functions above were allowed to be called in cells, then a recalculation of the spreadsheet could result in multiple instances of a *CostCalc* function changing the value of a cell at the same time. Hence, subroutines are key in ensuring proper interfacing between the spreadsheet and functions which alter the value of cells.

Additionally, subroutines can be utilized to performing repetitive operations in the spreadsheet, thus eliminating lots of manual inputting and saving time. In particular, subroutines were developed to cycle through a user-defined set of extrusions using a For Loop (incrementing some specified dimension or structural metric), each time inputting the current values into the *Dynamic Beam* table, and extracting the corresponding cost from the “Model” tab, as described below.

PerformBucklingCostCalculations() This Subroutine performs a sequence of cost calculations on an array of Euler buckling load constraints via *CostCalcBuckling* and

pastes the results in a user-defined array of cells. It extracts all of its input information from a table in the “Strength Analysis” tab called *Buckling Cost Calculations*, as shown in Figure 15.

Figure 15: *Buckling Cost Calculations* and *Deflection Cost Calculations* input tables

Buckling Cost Calculations	
Cost per	unit
No. Calculations	16
Top Cost	a15
Top Force	s5
Computed Dimension	Diameter
Fixed Dimension Value	6
Material Name	Al
Perform Buckling Cost Calculations	

Deflection Cost Calculations	
Load Type	End
Cost per	unit
No. Calculations	16
Top Cost	a1247
Top Deflection/Load	s247
Computed Dimension	Diameter
Fixed Dimension Value	5
Constant	Deflection
With Value	0.05
Material Name	Al
Perform Deflection Cost Calculations	

These inputs include the unit of cost (per unit extrusion or per kg), number of calculations to be performed, location of the top cell of the array where costs are to be pasted, cell location of the top Euler force constraint, dimension to be computed by *BucklingCalc* (thickness or diameter), the value of the fixed dimension in mm, and the material (Al or Mg). The subroutine is essentially a For Loop which, for each iteration, takes the current force constraint from spreadsheet, calls *CostCalcBuckling()* (thereby implicitly computing the thickness or diameter required to achieve the current force), and pastes the corresponding value in range of cost values on the same row as the current force. The loop then moves down to the next Euler force constraint and repeats the operation, iterating until an index has reached the input number of calculations. In this way, the user can quickly compute the cost of achieving a whole range of Euler buckling loads by varying either thickness or diameter.

PerformDeflectionCostCalculations() This subroutine is structurally identical to the previous one, except that it calls *CostCalcDeflection* within a For Loop and thus needs a slightly different set of inputs. An analogous *Deflection Cost Calculations* table, also shown in Figure 15, is used to provide the subroutine with these inputs. They include the

same parameters as used in *PerformBucklingCostCalculations* with a few extras – the type of loading condition (center or end), the deflection parameter held constant (either percent deflection or load applied in kN), and the value of that constant parameter. Specifying whether deflection or load is held constant is necessary so that the subroutine knows which parameter is to be iterated within the For Loop. Accordingly, the cell location specified as “Top Deflection/Load” may refer to the top of an array containing either percent deflections or applied loads in kN. As before, the user has the option of varying either the thickness or diameter in meeting the deflection constraint.

3.5 Press Size Determination

The last feature developed for the MSL Extrusion model is a method for determining the minimum size (and thus least expensive) press capable of producing a given extrusion based on the data tables in the “Extrusion Data” tab. The function *PressSize* utilizes the analytical methods described in Section 2.1 to compute the extrusion force as a function of the reduction ratio, billet diameter, and empirically determined extrusion constant k .

PressSize() *PressSize* computes the force required to extrude the *Dynamic Beam* using Equation 2. Since the reduction ratio depends on the billet dimensions, the function cycles through press ensembles, arranged in order of increasing billet diameter, constructed from the data provided by Ube Machinery (the bottom table in “Extrusion Data”). Each time it computes the force requirement based on the press data contained in that column and checks to see if it is less than or equal to the current press size (in MN). If not, it loops to the next press ensemble and repeats the calculation. It continues until the condition is satisfied, thus returning the least expensive press suitable for the extrusion. If a satisfactory press is not found, the function will return an error.

It’s worth noting that while the table is arranged in order of increasing billet diameter, some diameters can be used on multiple presses, so there is an occasional drop in press size corresponding to a lower pressure press as you move to the right. However, since the theoretical extruding force scales with the reduction ratio, it will not change over press

ensembles that have the same billet size, and hence, the set of presses which may be returned by the function increases monotonically in price from left to right.

Figure 16: Drop-down menu for method of determining press size

Press Data		Press Size Method	Manual Override	PRESS_SIZE_METHOD
		Calculated	CCD & Wall Thickness PressSize() Manual Override	
Press Size	N/A		16.05 MN	PRESS_SIZE
Container Diameter			0.178 m	CONTAINER_DIA
Area			0.025 sq m	PRESS_AREA
Maximum Billet Length			0.81 m	MAX_BILLET_LEN
Average Billet Weight			36.33 kg	AVG_BILLET_WT
Runout Table Length			42 m	RUNOUT_TABLE_LEN
Extrusion Weight			44.69 kg / m	EXTRSN_WT
Press Cost	\$3,111,726		\$3,111,726 \$	PRESS_COST
Handling System Cost			\$1,479,197 \$	HANDLING_COST
# of Extrusions per work hour			34 # / hr	EXTRSN_PER_HR
Extrusion Dead Time			1.76 min	EXTRSN_DEAD_TIME
Specific Pressure			0 MPa	SPECIFIC_PRES

In contrast to all functions described hitherto, *PressSize* is called within the cost model itself, inside the *Press Data* table. A drop-down menu, shown in Figure 16, gives the user a choice of method in how to determine the press size. They have the option of selecting “PressSize(),” which uses *PressSize* to determine the appropriate press, “CCD & Wall Thickness”, which looks at the CCD and minimum part thickness to assign a press size, or “Manual Override.” It’s important to realize that “PressSize()” takes data from the bottom table in “Extrusion Data” (from Ube Machinery as explained in 2.2.2), while “CCD & Wall Thickness” takes its data from the pre-existing top table of press sizes. For “Manual Override,” the user must specify in the cell labeled ‘PRESS_OVERRIDE’ a valid press size which exists in the bottom table. This requirement is justified by the belief that the newer data from Ube Machinery is likely more accurate and reliable than the pre-existing data.

It’s worth noting that each in each iteration of either cost subroutine, *PressSize* recalculates along with all other formulas in the cost model. Thus it’s possible the press size will change in the middle of a subroutine. This is perfectly valid and would likely result in a distinct step up or down in corresponding cost plots. However, due to the large

differences in force between the presses contained in the table, it is highly unlikely to happen when only minor changes in dimensions are occurring.

Collectively, the mechanical functions – *BucklingCalc*, *DeflectionCalc*, *Deflection*, *Load*, and *Weight* – the cost functions and subroutines – *CostCalcBuckling*, *CostCalcDeflection*, *PerformBucklingCostCalculations*, and *PerformDeflectionCostCalculations* – and *PressSize* completely describe the features developed within the “Strength Analysis” tab of the MSL Extrusion model for systematically studying the relationship between strength, weight, and cost of extruded, semi-hollow, Al and Mg cylindrical rods. Having detailed in Section 2.3 some of the real-life applications of such extrusions, we now turn to a case study to demonstrate the capability of these functions in meeting this goal.

4 Case Study

4.1 System and Assumptions

The basic system under consideration in this case study is a 1 m long, Al or Mg hollow tube of wall thickness of 6-8 mm and diameter of 70-75 mm. This generic part shape was chosen because of its wide applicability to the various real-life loading scenarios described in 2.3.1 – 2.3.3. Indeed, many structural members in the space frame and chassis of an automobile fall within this general range of geometries, and while often not cylindrical, they can be reasonably approximated as so.

Moreover, we choose the same base case for both Mg and Al so as model the real-life situation where one is considering replacing an existing Al part with a similar Mg version. This is additionally helpful since it requires that cost differences between Al and Mg arise strictly from strength constraints and not from different initial geometries. Otherwise, it would be difficult to isolate these effects. Further, throughout this analysis, the length was kept constant at 1 m while thickness and diameter were allowed to vary around their base values within the various cost and mechanics functions. This is justified by the fact that in reality, the geometry of an automobile strongly constrains the length of

various parts, whereas the thickness and diameter can be varied somewhat without greatly affecting the space constraint.

Table 4: Key material dependent assumptions

Parameter	Al Value	Mg Value	Explanation
Extrusion Rate [m/min]	19	15-25	Similar extrudability values, consistent with ram speeds of 15-25 mm/s [14]
Extrusion Temperature [°C]	400	394	Constant percentage of melting temperature [9]
Extrusion Constant [MN/m ²]	68.9	13.8	Derived in section 2.1
Press Cost [\$ / US Ton]	\$1150	\$1725	Manufacturer's data for Al [6], Mg roughly estimated to be 50% more expensive
Billet Price [\$ / kg]	\$2.98	\$3.53	Pre-existing data, Mg more expensive as expected
Press Scrap Price [\$ / kg]	\$2.00	\$1.77	Bulk Mg scrap worth less than Al due to lower demand
Fabrication Scrap Price [\$ / g]	\$1.63	\$1.77	Post-extrusion Al scrap worth less than bulk scrap, not as significant for Mg

In addition to part geometry, a whole host of assumptions were made for various cost model inputs. Key material-dependent parameter assumptions are listed in Table 4 above. While these assumptions are all fairly intuitive, extrusion rate deserves some additional mention due to its effect on cycle time and particularly strong capacity as a cost driver. While literature often defines extrusion rate in terms of ram speed, the cost model defines it in terms of the runout table speed which is considerably faster due to the reduction of cross-sectional area. For reduction ratios in the range 10-20, literature ram speeds for Al of 12.7-25.4 mm/s are consistent with constant runout speeds of ~8-30 m/min. Hence, the middle value of 19 m/min was used as the Extrusion Rate for Al. This value was used as the baseline for Mg as well since Al and Mg have roughly the same extrudability; however, higher and lower values were used for a sensitivity analysis to account for real-life variation. In general, ram speeds are generally related to hardness, with Al and Mg being extruded much slower than harder steels.

There are also many exogenous inputs into the cost model which are material-independent, the most important of which are enumerated in Table 5. Again, most of these inputs are intuitive and do not warrant additional explanation. However it's worth mentioning that the scrap and reject rates for all steps of the process were set to a

reasonable value of 1% for sake of simplicity with the exception of the scrap rate of the first step Billet Preparation. In this step, the scrap rate is not an input, but rather is implicitly defined by the integer number of parts which can be extruded from the billet. Whatever is left goes unused in the billet butt and is scrapped at the press scrap price. For all steps after extrusion, scrapped material is sold at the slightly lower fabrication scrap price, reflecting the fact that it's harder to recover usable material from fabricated parts than from bulk. In general, the effect of the 1% scrap and reject rates is to reduce the weight and number of parts leaving each successive step of the extrusion process. Based on the annual production volume and final part weight, the model can thus use these rates to back calculate the number quantity and unit weight needed to enter each step, going all the way back to first step and thus defining the initial number of billets needed.

Table 5: Key material independent assumptions

Parameter	Value	Explanation
Annual Production Volume [parts/yr]	40,000	Reasonable value corresponding to ~170 extruded parts per working day
Direct Wages	\$25.00 / hr	High end for skilled labor and associated management
Press Size	16.05 MN	Determined by <i>PressSize</i> for system under consideration
Unit Electricity Cost	\$0.07 / KW-hr	Time-averaged market price
Interest	10%	Standard rate for amortization of plant and equipment
Equipment Life	20 yrs	Amortization period of extruder
Indirect/Direct Workers	0.25	Reasonable estimate for additional indirect labor incurred
Building Unit Cost	\$1500 / m ²	Reasonable cost of floor space
Building Life	40 yrs	Amortization period of building
Production Life	5 yrs	Amortization period of cutting tools
Idle Space	25%	Reflects empty space in plant
Working Capital Period	3 months	Reasonable value to ensure adequate operational liquidity
Heating Efficiency	40%	Conversion of electricity to heat
Heat Loss Factor	50%	Heat lost to environment
Heat up / Billet Dwell time	200 s / 800 s	Reasonable time to heat up billet
Extrusion Power	20 kW	Power requirement of extrusion press
Downtime	5%	Fraction of time production is halted
Scrap/Reject Rates	1%	All manual inputs set to 1% for all steps except Billet Prep, where scrap rate is determined by no. of parts in billet
Scrap Recovery Rate	40%	Only a fraction of scrap is actually sold at scrap price
Aging Duration	10 hrs	Annealing time ensures strong crystal structure
Handling System Cost	\$820 / US Ton	Manufacturer's data assumed to be equal for Al and Mg [6]

Finally, it's worth noting that a constant Press Size of 16.05 MN was used throughout the following analysis. For the Al and Mg systems in question (including their dimensional extremes), this is the value determined by the *Press Size* function described in Section 3.5. Having addressed all of the inputs used in the cost model, we can now demonstrate

an analysis of the system described above using the functions and interface described in Section 3.

4.2 Sensitivity to key input parameters

Prior to analyzing various loading scenarios, it's useful to understand general cost sensitivity to some important model inputs, specifically extrusion rate, billet and scrap price, and part thickness and diameter. Further, these sensitivities will be explored here and later on for *Magnesium only*, since the goal here is consider the aspects of a potential Magnesium process which make it preferable over *some baseline Aluminum process*.

Later on we shall relax strength, geometric, and processing constraints on Mg to investigate how these sensitivities affect its desirability over Al. This analysis thus seeks to simulate the real-life scenario where a manufacturer has a pre-existing, effectively *non-modifiable* Al process and is considering whether or not a switch to a *new magnesium process*. This includes both replacing the existing process with Mg and creating a new Mg part with modified dimensions. This is realistic in the sense that a manufacturer would only switch to Mg if weight savings existed, and that if they could gain by modifying the aluminum process, they would have already done so.

To this end, we will stick to the base system described in Section 4.1 for Aluminum. The following sensitivity analyses for Mg will prove useful in addressing more complex loading scenarios later on.

Figure 17: Extrusion Rate Sensitivity

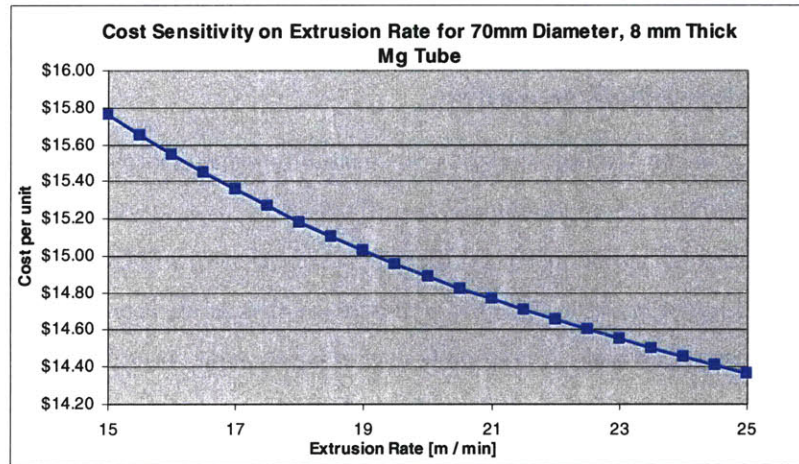


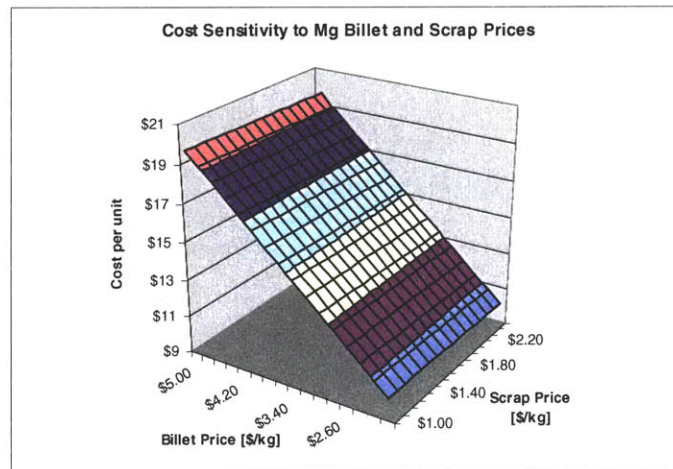
Figure 17 illustrates how the cost of extruding a standard 70 mm diameter, 8 mm thick Mg tube varies with extrusion rate. All else held constant, the cost can be reduced ~4.5% by increasing the extrusion rate from 15 to 25 min/min. This results from the fact that all time-dependent quantities in the cost model – cycle times, labor costs, energy costs, etc – all fall when extrusion speed is increased. Further, this range of costs falls well below the \$17.77 cost of making the exact same part in Al at 19 m/min. That is due to the fact that despite Mg being ~20% more expensive, the ~2/3 density means less material is needed, so overall cost is lower.

However, due to the reduced modulus and yield strength of Mg, this part will be much weaker than its Al equivalent in every loading scenario, meaning that additional volume (and hence mass and cost) will be needed to achieve an equivalent strength constraint as Al. While subsequent analyses will keep extrusion speed constant at 19 m/min, in practice there is some flexibility to extrusion speed depending on grain structure, composition, and hardness, so this is useful to know.

Figure 18 illustrates cost sensitivity to Mg billet and scrap prices. As one would expect, cost is strongly dependent on raw billet price, nearly doubling as the raw billet price increases from \$2 to \$5/kg. This results from the fact that raw material costs account for

~70% of the total cost of extrusion. Interestingly, unit cost is only weakly dependent on scrap price. This is due to the fact that a small percentage, usually 5% or less, is lost as scrap in the butt, and of that only 40% is recovered. Further, scrap prices are roughly half that of the billet price, so in the end, the scrap credit does little to effect the final cost.

Figure 18: Cost Sensitivity to Mg Billet and Scrap Price



A side point is the strong dependence on billet price means that cost effectiveness of Mg is highly dependent on current market prices. If Mg price was lower and scrap was higher, both of which might result from increasing competition and demand in magnesium supply industry, then this may be very relevant to a manufacturer. However, movements in Mg and Al prices are likely to be correlated, and no doubt Mg will continue to remain more expensive in the near future. For this reason, we stick to the billet and scrap prices listed in Table 4 throughout this analysis, but nevertheless, this sensitivity is useful to understand for future studies when prices have changed.

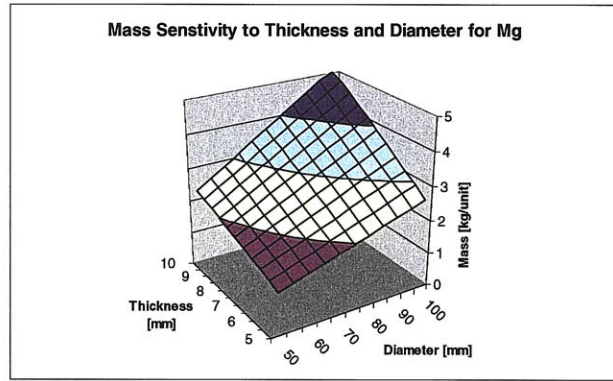
The final sensitivity to address is that of part dimensions – thickness and diameter - on mass. Having established above how strongly cost depends on billet price (and mass), it's useful to understand how quickly, or equivalently volume, moves with changes in these dimensions, as illustrated in Figure 19 for a length of 1 m. As clearly seen, mass increases much faster with increasing thickness than diameter.

This is due to the fact that, for a tube of diameter 70 mm and thickness 6mm,

$$\frac{dV}{dD} = \left(\frac{t}{D-2t} \right) \cdot \frac{dV}{dt} = \left(\frac{6}{58} \right) \cdot \frac{dV}{dt} \quad (10)$$

This means that volume (and hence mass) increases ~10X faster with increasing thickness than an equivalent increase in diameter. From a purely geometric standpoint, this analysis shows that if strength or geometric constraints require thickness to change, there will larger cost implications than if constraints are keeping thickness fixed and varying diameter.

Figure 19: Mass Sensitivity to Thickness and Diameter



4.3 Axial Loading and Cost Implications of Euler Buckling Load Constraints

The first loading condition considered is that of axial loading under an Euler buckling load constraint. Figure 20 illustrates the Mg and Al cost curves for 1 m long tubes of constrained 70 and 75 mm diameter. Constrained diameter is an appropriate model for real-life applications where the geometry of the vehicle places package constraints on the components. It's important to realize that at each point, *BucklingCalc* has implicitly computed the thickness necessary to achieve the desired MN load; hence, thickness is implicitly increasing moving left to right. Figure 21 shows the corresponding cost and weight changes associated with switching from Al to Mg at each value on the x-axis. Throughout this analysis, a cost or weight change is defined as the value of magnesium minus the value of aluminum.

Figure 20: Comparison of Cost to Axial Strength – Constrained Diameter

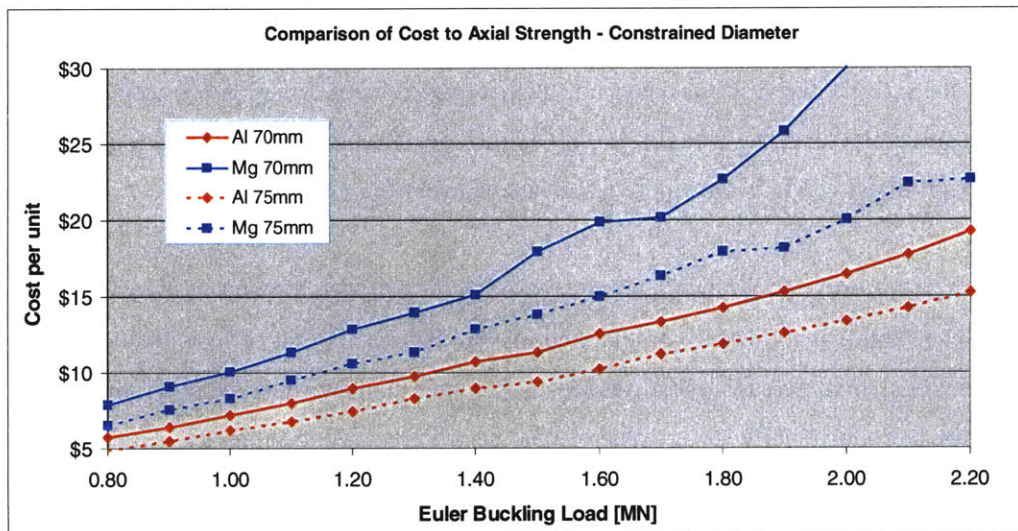
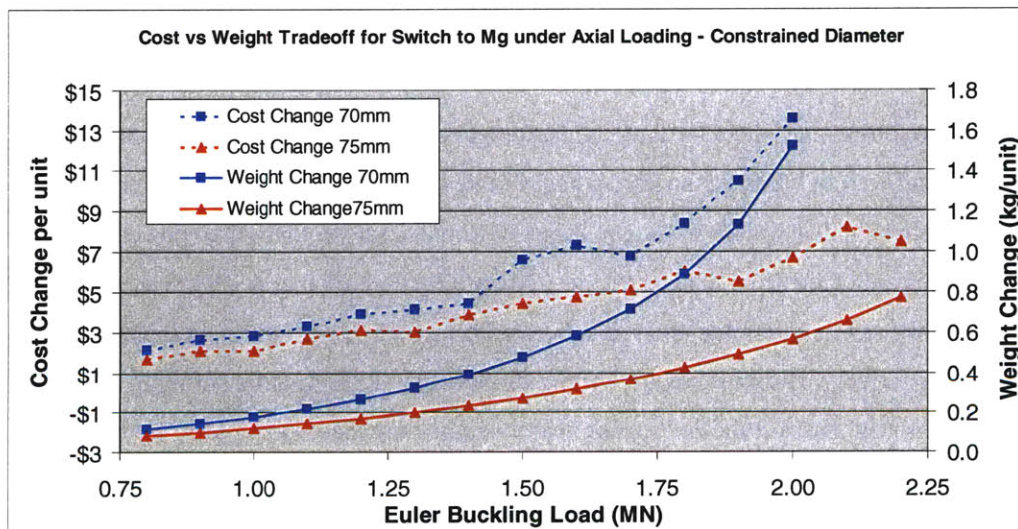


Figure 21: Cost vs. Weight Tradeoff for Switch to Mg under Axial Loading - Constrained Diameter



We can see from these figures that Mg is more expensive than Al at both 70 and 75 mm diameter. This is primarily due to the fact that Mg has to compensate for its lower Young's modulus by attaining a higher area moment of inertia. However, since cross sectional area moves more strongly with outer diameter than thickness, Mg must increase

it's thickness by so much when its diameter is constrained that it ends up being much heavier than the Al strength-equivalent, as reflected in the solid curves of Figure 21. Indeed, the ratio of derivatives is the same as that expressed in Equation 10, noting that volume and cross-sectional area differ only by the constant factor of length. Coupled with a higher billet price, Mg is thus more expensive at every buckling load.

However, it's worth noting that the net increase in cost between Mg and Al *decreases* from 70 to 75mm, as reflected by the dotted lines in Figure 21. This reflects the fact that cross sectional area increases more quickly in the outer diameter than the inner diameter. At 75mm, a proportionally lower value of thickness is required to achieve a certain cross sectional area than at 70mm, meaning that there is a net reduction in weight (and hence material cost) moving to 75mm. At 2.00 N for instance, the 70mm Mg tube is roughly 1.5 kg heavier than its Al counterpart, whereas at 75mm it is only ~0.6kg heavier. In general, for any particular strength, we can continuously decrease weight by increasing diameter and reducing thickness until we reach the geometric limits of either the application or the extruder itself.

This is a troubling outcome – holding axial strength and diameter constant, we increase *both* cost and weight when we switch to Mg. In this scenario, Mg would never be desirable. However, if we are willing to sacrifice some axial strength, then one can imagine attaining a positive weight savings, potentially making it cost effective. From Figure 20, we see that the blue and red solid curves cross \$10 per unit at ~1.0 MN and 1.3MN respectively. Hence if we are willing to sacrifice ~23% of axial strength, we can make 70 mm Mg and Al tubes of approximately equal cost. Sacrificing less, we will find a region where Mg costs more but also exhibits some weight savings. Sacrificing more, we can make Mg both cost and weigh less, as shown in Figure 22.

Here, the curves represent the differences in cost and weight between an Al tube and Mg tube of exactly 1 MN lower Euler buckling load. We see that such a large strength sacrifice allows us to save both on both cost and weight when switching to Mg. When switching from an Al tube of 2.2 MN strength to a Mg tube of 1.2 MN strength, we save

2.07 kg weight and \$6.39 in cost at 70 mm. Obviously most applications, such as primary members in the space frame body structure or other structural applications, will not tolerate such a large sacrifice of strength; however, if such a large strength is not critical from a safety or function perspective, then it's certainly worthwhile to consider. Alternatively, sacrificial Mg parts could be used in cars specifically for the purpose of buckling and absorbing energy in a collision, thus buffering more critical structural members. Of course, one could sacrifice strength by simply reducing the dimensions of aluminum – that would indeed lower its cost as well. However, the idea here is that if there already exists a desire to switch to Mg for *specific strength* gains, then sacrificing strength is a way to make it cost effective.

Figure 22: Cost vs Weight Tradeoff with 1 MN Axial Strength Sacrifice

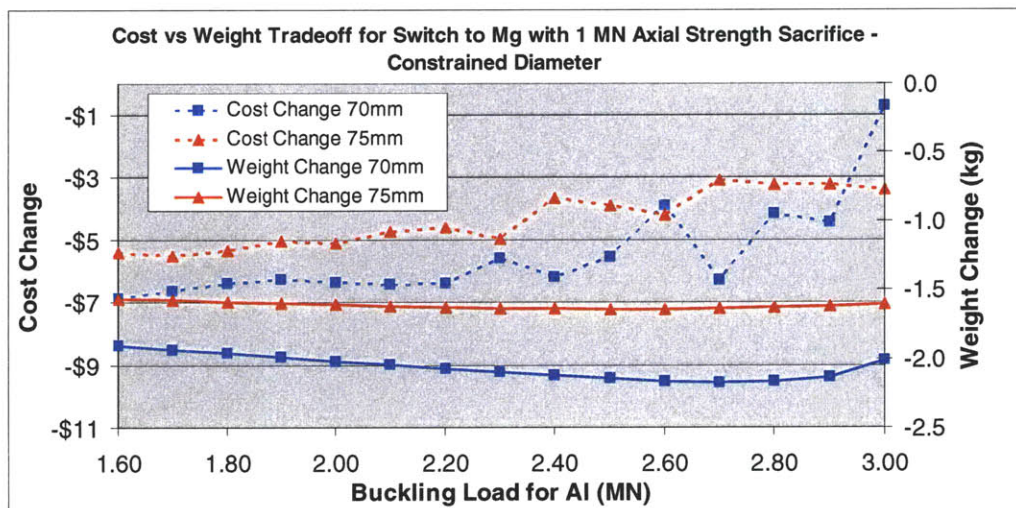
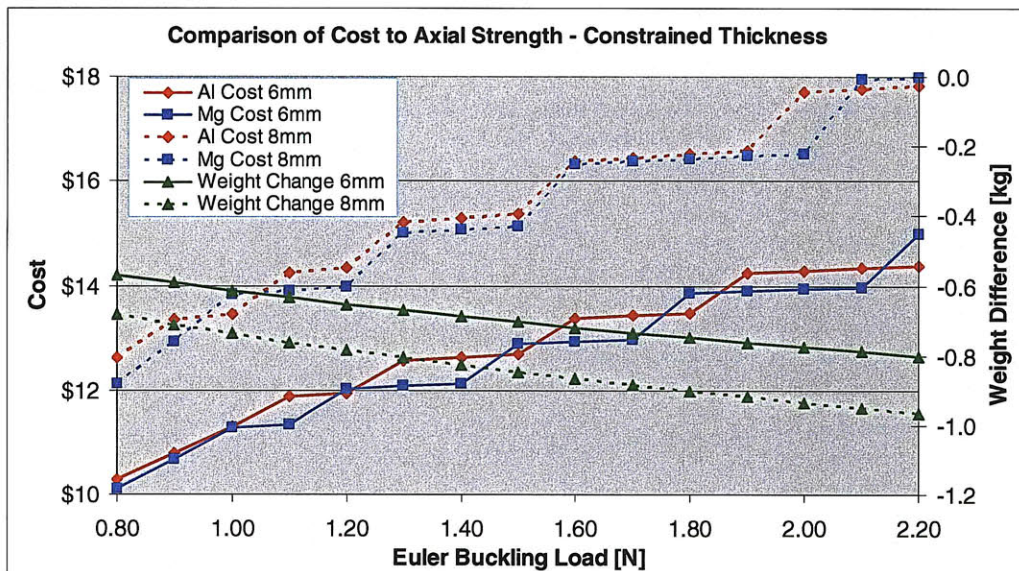


Figure 23 depicts the opposite case - that of constraining thickness and letting the diameter vary. This constraint is applicable to situations where a part is not package constrained, such as exposed parts or those not directly contacting other components along their length. The first obvious difference is that the Mg and Al cost curves at both 6 and 8mm track each other quite closely, as opposed to Mg always being more expensive. This interesting phenomenon arises simply because of the particular values of modulus and density. Having a lower modulus, the diameters computed by *BucklingCalc* are

~15% higher for Mg than for Al; however, since the density is ~2/3 of Al, the weight is always less by ~23%. However, the ~17% higher billet price of Mg roughly cancels out this effect, thus allowing the cost curves to track each other quite nicely at both 6mm and 8mm. This general effect holds even at higher thicknesses of 10-15mm. The stair-step shape of the curves is a result of the discrete jump in price associated with part mass increase such that the integer number of parts attainable from the billet decreases by one. When this happens, there is a simultaneous increase in the number of billets needed, the amount of material lost in the butt, and the annual production time required.

Figure 23: Comparison of Cost to Axial Strength - Constrained Thickness



Secondly, it's notable that the cost curves for 8mm are substantially higher than at 6mm. This again arises from the fact that the area moment of inertia depends more strongly on outer diameter than thickness, such that the net weight increase associated with the 2mm increase in thickness outweighs the effect of needing a slightly lower diameter to meet some strength constraint. For example, *BucklingCalc* shows that Al needs a diameter of 59.33 mm to achieve 1.00 MN buckling load at 6mm thickness, while at 8mm thickness, the diameter required drops to only 56.22 mm. The increase in thickness from 6 to 8mm

increases the volume much more than the reduction in diameter reduces it, thus leading to higher mass and material costs.

The fact that the area moment (and hence buckling load) moves so strongly with outer diameter has major implications on weight. When switching to Mg, you need a larger diameter to offset the lower modulus, but since it moves so strongly, you need not increase it by that much. As a result, when switching to Mg, the ~33% lower density dominates the marginal increase in volume associated with expanding the diameter, thus resulting in mass reduction as indicated by the green plots in Figure 23. At 6mm, switching to Mg results in a mass reduction of 0.57-0.8 kg, while at 8mm, it nearly reaches 1.0 kg. This mass reduction comes at almost no cost expense, since as described above, the costs curves track each other very closely, sometimes with Mg cheaper and other times Al. Hence, under constrained thickness, it is always advantageous to switch to Mg and save weight assuming that prices are such that production costs remain similar.

4.4 Constant Center Loading with Variable Deflection

The next loading condition is that of center loading. Figure 24 illustrates cost vs. deflection curves for Al and Mg tubes of constrained diameter under a 20 kN center load. This scenario can be used to model situations in which a known force is applied under a deflection constraint, for example computing the dimensions needed to ensure that the force of a 20 MPH side collision doesn't deflect the space frame by more than 5%.

Moving to the right, *BucklingCalc* computes an implicitly decreasing thickness necessary to achieve a given deflection under the constant center load. Since cost tracks volume, cost decreases as well. Similarly to the behavior for axial loading, magnesium is again more expensive for both diameters, and further, both metals are cheaper when constrained at 75mm than at 70mm due to the strong dependency between the area moment of inertia and outer diameter.

Figure 24: Cost vs. Deflection under 20 kN Center Load – Constrained Diameter

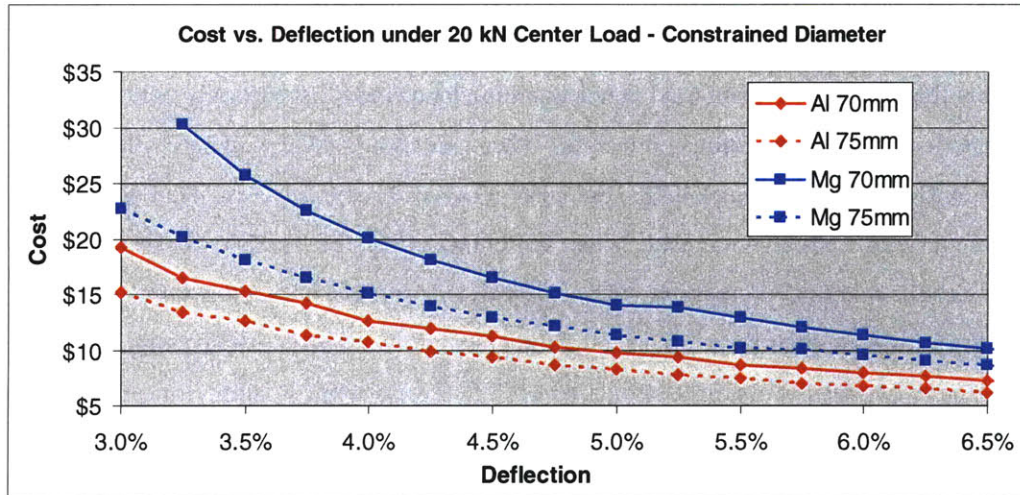
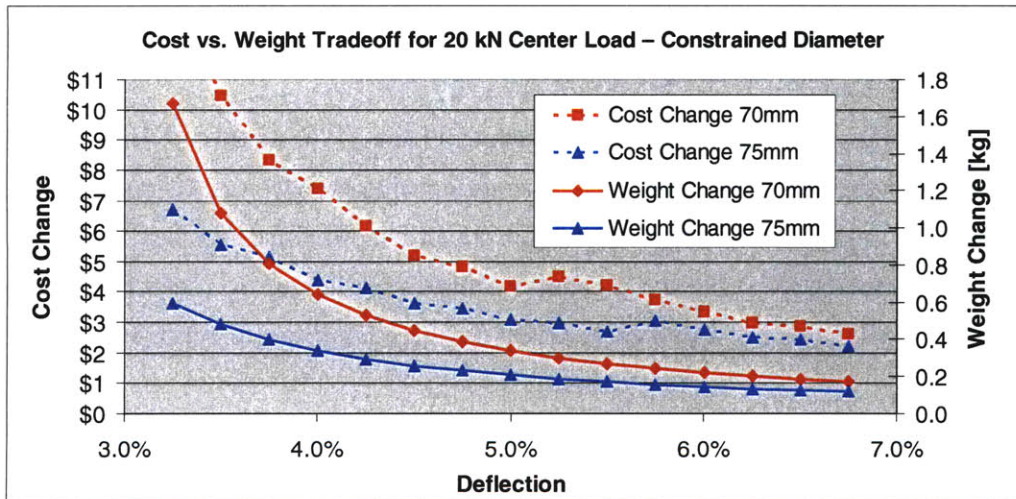


Figure 25: Cost vs. Weight Tradeoff for 20 kN Center Load – Constrained Diameter



As shown in Figure 25, the cost change associated with switching to Mg decreases moving out to higher deflections due to the convergence of the implicit Mg and Al thicknesses. However, as before, there is a weight gain associated with the switch to Mg due to the relatively high thicknesses required to achieve the given constraints. While this weight gain becomes very small at high deflections, in reality one would not want to sacrifice so much strength, especially if it results in a weight *gain* instead of a weight

loss. Indeed, without relaxing some constraint in Mg, either diameter, extrusion rate, or strength, there are no feasible scenarios when switching to Mg is desirable under constrained diameter center loading.

Figure 26: Comparison of Cost to Deflection under 20 kN Center Load – Constrained Thickness

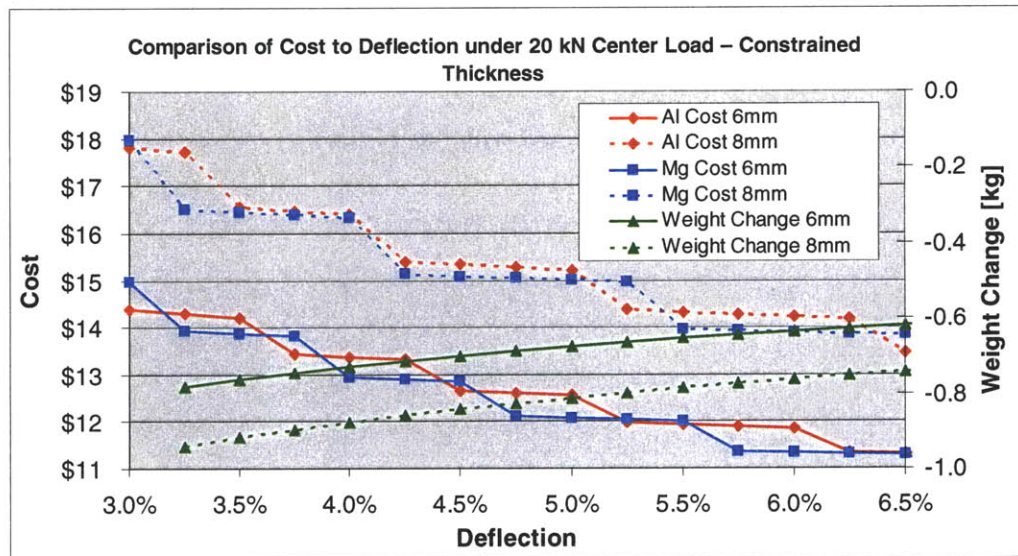
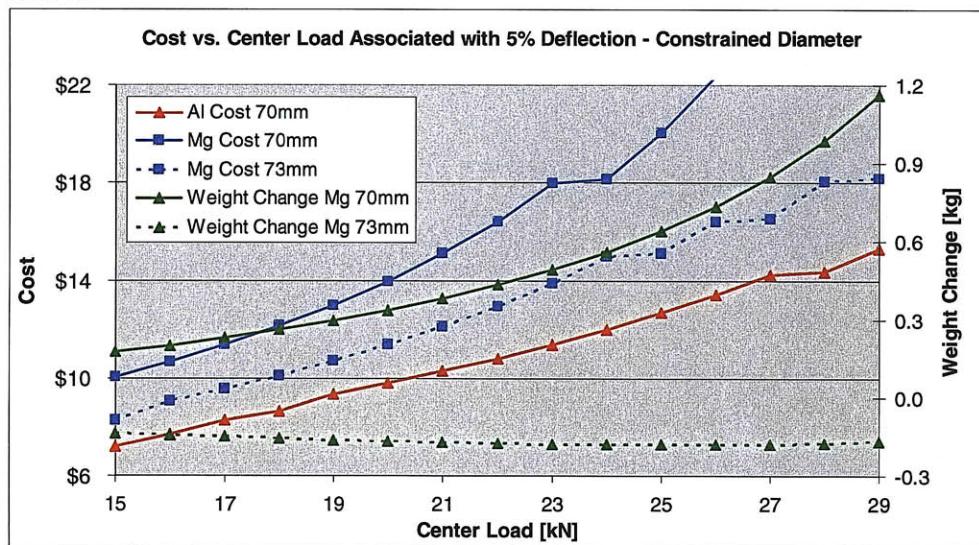


Figure 26 illustrates the same loading scenario but with constrained thickness of 6 and 8 mm. As with axial loading, the cost curves at both thicknesses track each other quite nicely, resulting in a small difference in cost. At any particular deflection, which one is more expensive depends mostly on the integer number of parts attainable in the billet, which is why, as volume changes, it alternates back and forth. Again there is a weight savings associated with switching to Mg due to relatively small changes in diameter needed to achieve a higher moment and thus a higher strength. Hence, when not package constrained, such as with parts that have free space around them, switching to Mg is practical from both a cost and weight perspective. In reality though, it could be that diameter is semi-constrained - that is, it can vary within some narrow range but not freely as is the case with constrained thickness. This intermediate case is addressed in the next section which describes the opposite loading scenario - modeling a constant deflection under varying center load.

4.5 Constant Deflection under Varying Center Loads

In this scenario, *BucklingCalc* computes the dimensions corresponding to some constant deflection under varying center load, as illustrated in Figure 27 for the case of 5% deflection and constrained diameter. This could be used, for example, to model the real-life 5% deflection threshold for a spectrum of side collision forces.

Figure 27: Cost vs. Center Load Associated with 5% Deflection – Constrained Diameter

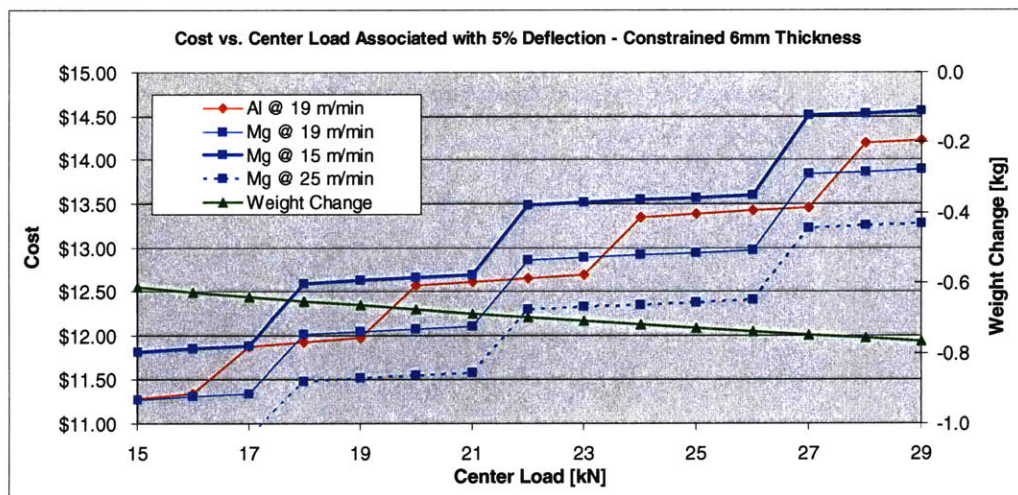


As expected from earlier constrained diameter scenarios, 70 mm Mg is not only more expensive than 70 mm Al, but also weighs more over the entire spectrum of loads, as illustrated by the solid blue, red, and green plots respectively. However, knowing that both factors can be reversed to some degree by letting diameter increase, it's natural to ask: what if we let only the Mg diameter vary slightly? Is it possible that a switch will become advantageous? This 'semi-constrained' scenario reflects the fact that in reality, a part shape may be slightly modified if switching to Mg is deemed feasible.

As we'd expect, the Mg cost curve decreases if the diameter constraint is pushed out to 73mm. However, what's more interesting is that by simply increasing the constraint by

3mm, the weight gain turns into a *weight loss*, as depicted by the dotted green curve. So while it's never advantageous to switch if Al and Mg are held to the same constraint, Mg quickly becomes preferable if we have some flexibility to expand the diameter slightly. By similarity of functional forms, these conclusions would also follow for axial and end loading conditions under constrained diameter. Of course, one might ask: why not simply make Al at 73mm? Obviously this would be cheaper than making an identical Mg tube, but as described earlier, the focus here is not whether Mg is preferable over an adjustable Al process, but rather to discern which Mg scenarios make it preferable over some pre-existing, constrained Al process.

Figure 28: Cost vs. Center Load Associated with 5% Deflection – Constrained Thickness



Having explored geometric flexibility, what if instead Mg extrusion rate changes? Indeed, in reality there is some uncertainty about what the actual proper extrusion speed will be due to differences in the microstructure, composition, processing conditions, and relative hardness of the raw material. Figure 28 illustrates the same loading constraint as Figure 27 but with constrained thickness and variable Mg extrusion speed. As we saw earlier, constraining thickness allows for a reduction in weight by switching to Mg with little or no extra cost at the same extrusion speed of 19 m/min.

However, if Mg extrusion speed is slower or faster, as might be the case, then Mg becomes absolutely more or less expensive, as depicted by the bolded and dashed blue lines respectively. Indeed, at 25 m/min, Mg is both cheaper and weighs less than its Al counterpart for the entire range of center loads examined. However, for constrained diameter, one would not even bother hoping for a faster extrusion speed despite the reduction in cost since the Mg equivalent still weighs more – one would simply choose to stay with Al. Nevertheless, these previous two examples demonstrate how the relaxation of geometric (diameter) or processing (extrusion speed) constraints expand the number of scenarios in which switching to Mg is advantageous.

4.6 Constant End Loading with Variable Deflection

Figure 29: Cost vs. End Deflection – Constrained 70 mm Diameter

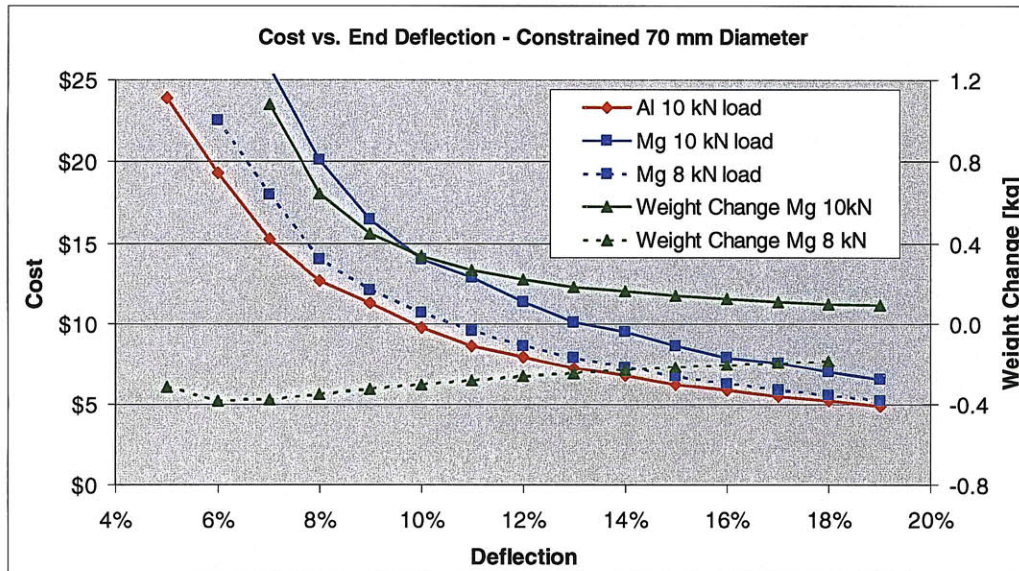


Figure 29 is the end loading analogue of Figure 24 for a constrained diameter of 70 mm. First notice that the deflection range of 5-19% extends much further than the 3-6.5% range of Figure 24. This is despite the fact that lower loads (8 & 10 kN vs. 20kN) are being applied. This is a direct consequence of the fact that a beam is weaker and deflects more in end loading than in center loading. Like before, switching to Mg at 10 kN not

only results in an increase in cost but also a weight gain, thus making the switch unfavorable when diameter is constrained.

However, in the spirit of relaxing constraints, observe what happens when we reduce the load 20% to 8 kN, still maintaining a 70 mm diameter. As shown by the dotted blue curve, the cost of switching to Mg drops 20-30% depending on deflection, making it only slightly more expensive than Al at 10 kN. More importantly though, the switch from Al 10 kN to Mg 8 kN results in a *weight loss*. If strength was sacrificed slightly more, it's conceivable that Mg cost would actually become cheaper than Al 10 kN. While sacrificing strength sounds inconceivable, it might actually be feasible in situations where the manufacturer is considering replacing structurally non-critical components, or rather when making a lighter car, which by virtue of weighing less, needs less strength to maintain its safety. In other words, specific strength, that is strength divided by density, might be more important in some circumstances than absolute strength.

Figure 30: Cost vs. Deflection Associated with 10 kN End Load – Constrained 6mm Thickness

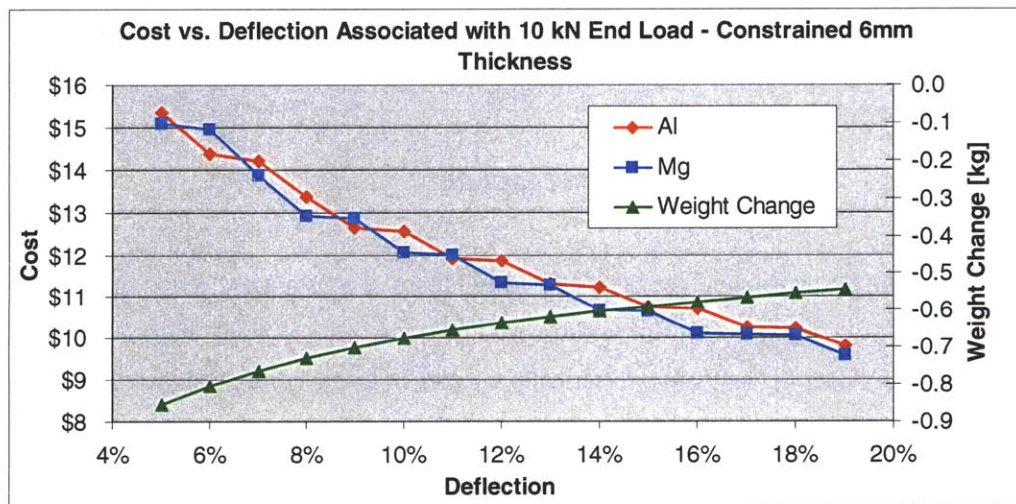
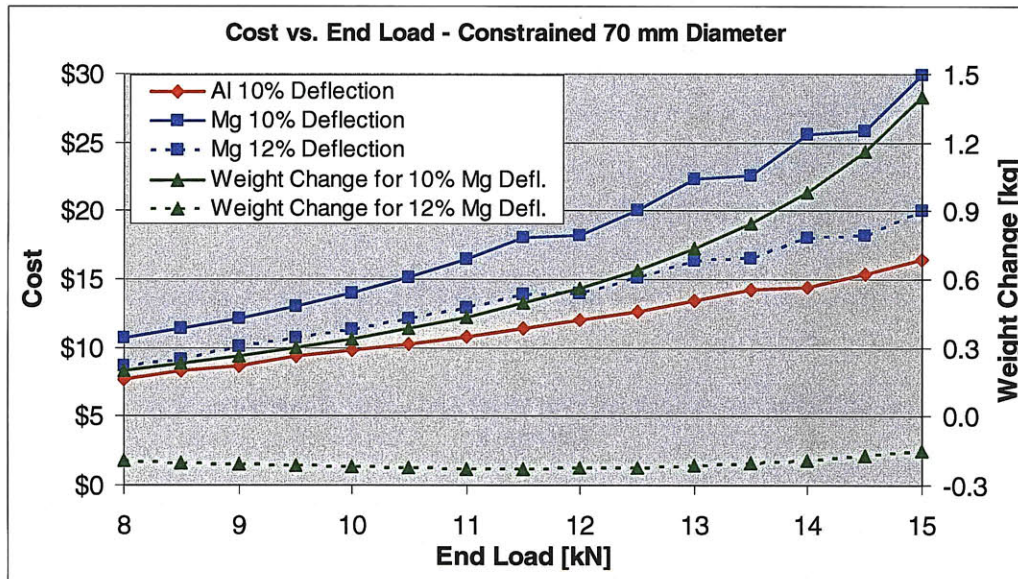


Figure 30 presents the same loading scenario but instead with a constrained thickness of 6mm. Having already established in Figure 26 the effect of higher thickness, that effect is omitted here. Indeed, the data behaves identically to the center loading scenario, with Al and Mg cost curves tracking each other closely and having a switch from Al to Mg

associated with a weight loss. Accordingly, if diameter can vary due to lack of package constraints, switching to Mg is preferable, especially if Mg can be extruded faster as shown earlier in Figure 27. The only main difference between this scenario and Figure 26 is that deflection constraints are much more costly to meet in end loading than in center loading at 6mm thickness, as one would expect.

4.7 Constant Deflection under Varying End Loads

Figure 31: Cost vs. End Load – Constrained 70 mm Diameter



The final loading scenario analyzed is that of maintaining a constant deflection under varying end loads. Figure 31 illustrates the condition under a constrained diameter of 70mm. With end loading being a mechanically weaker state, we consider lighter loads (8-15 kN vs. 15-29 kN) and greater deflections (10% and 12% vs. 5%) than we did with center loading. Moreover, meeting a deflection constraint is much more costly for both Al and Mg due to the additional volume required. Like in Figure 27, it becomes progressively more expensive to maintain constant deflection under increasing loads, with Mg being more expensive than Al and also weighing more. Switching to Mg is clearly not favorable under constrained diameter unless we relax some constraint. As shown in the dotted plots in Figure 30, relaxing the deflection to just 12% results in a 19-

33% reduction in cost and a *weight savings* of up to 0.23 kg over Al at 10% deflection. Relaxed further, these gains would no doubt increase. Hence, if an application can conceivably allow for small sacrifice in strength, Mg can quickly become competitive and even preferable to Al. Of course, we could make these same relaxations on aluminum and reduce its cost, but again, the focus is on making magnesium preferable to some *baseline aluminum process*. The final analysis would be that of constant thickness, but this presents no new information over Figure 30, so we will not consider it here.

4.8 Case Study Summary

In general, when thickness is constrained and diameter allowed to vary, switching to Mg results in a weight reduction with little or no extra cost. This benefit can be enhanced by increasing the extrusion rate or relaxing the strength constraint on Mg. This is true for all three considered loading conditions. By contrast, when diameter is constrained, Mg is never preferable unless some constraint is relaxed, which is indeed conceivable in some real-life applications.

With reduction in weight come lower forces in collisions, so lower strength constraints may be feasible. Perhaps the part is not a critical structural element, or perhaps we are designing a sacrificial part *designed* to fail or deflect in a collision. In other words, *specific strength* might be more important to the manufacturer than *absolute strength*. Additionally, in semi- or non-package constrained situations, increasing diameter may be an option for some parts without causing the car body and frame to be redesigned. In general, when facing some strength constraint, the cost and weight of Al and Mg hollow extrusions can always be reduced by making a thinner, larger diameter part; however, the degree to which this is feasible depends entirely on the package constraints imposed by its particular application.

5 Discussion

While this case study is undoubtedly insightful, its results must be considered within the context of several important limitations. These limitations arise because of the model's

incredibly strong dependency on a few key cost drivers, the first of which is material cost. Accounting for ~70% of overall cost, material cost and hence market prices of Mg and Al are probably the single most important movers of the cost curves considered in the case study. On one hand, these prices can shift substantially over time, both in absolute and relative terms, so it's incredibly important that anyone hoping to acquire useful results use current prices which accurately reflect their cost of production. However, it's important to note that through the use of futures and forward contracts, manufacturer's can lock in material prices and hedge against price increases. In this sense, using constant prices in the model, as we have done here, is a fair assumption for a general analysis given that they are reasonable in current market conditions.

Secondly, as the second largest component, labor cost is critically dependent on various assumptions of factory labor structure, such as wages, the number of workers per line, the ratio of indirect to direct labor, line uptime and downtime, etc. While kept constant in this analysis, in reality these values can change over time depending on economic conditions, plant reorganization, and increasing efficiency via 'learning' in the production process. Further, assuming the use of Mg involves the installation of new machinery and processes, the values describing labor structure in Mg extrusion may differ from Al. However, given the limited information known about the Mg process, this assumption of equality was certainly a good one for this analysis, and results will only improve with better inputs.

The range of reasonable values for various extrusion parameters - extrusion speed, billet size, and press cost - also expands the confidence interval for the results of the cost model. Extrusion speed ultimately depends on the relative hardness of the material and die, so while we have chosen reasonable speeds for Al and Mg this analysis, in reality they can vary over some range depending on microstructure and composition. Further, via the *PressSize* function, the 16.05 MN press and its associated billet was kept constant throughout. However, in reality a manufacturer could choose to use a larger, more expensive press with longer billets, which would have the effect of reducing scrap waste and the number of extrusions needed to meet the production volume. Additionally, a very

rough estimate was used for Mg press cost. Amortized over 20 years, press cost accounts for 3-5% of total cost, so when better Mg press data is available, it's possible there could be a slight effect on Mg cost one way or the other. It's worth mentioning that softer Mg can be extruded at lower pressures than Al, hence requiring less power. However, this difference will be relatively small, and, coupled with the fact that energy accounts for ~1% of total cost, the implications on overall cost are minor.

Having explored limitations of interpreting cost behavior, it is also important to consider aspects of the strength analysis which may deviate from reality. Most importantly, here we have compared the mechanical performance of pure Al and Mg, whereas in reality, Al-Mg alloys of various compositions would be used for various reasons. First of all, alloying allows manufacturer's to take advantage of Mg's light-weighting capabilities while still taking advantage of aluminum's strength and inertness. While work hardening often provides additional strength, pure Mg is hardly ever used in demanding applications such as a structural member in a vehicle due to its relatively low strength and high reactivity [14]. While a surface oxide layer partially masks this reactivity, it is inferior mechanically and chemically to the Al-Mg alloys which dominate industry. Moreover, using alloys creates an easier transition for the production process and provides a manufacturer with an opportunity to sample Mg properties without full commitment.

The major implication of this is that there are plenty of intermediate scenarios between the extremes presented in the case study. Depending on composition, Young's modulus, billet price, and density can take on an entire range of values, meaning that in addition to the geometric, processing, strength constraints relaxed in the analysis, one can also alter composition in cost comparisons. By simply adding a new material to the "Material Data" tab of the spreadsheet, one could easily perform all the analyses described in the case study with an Mg-Al alloy, potentially altering some results while introducing insightful new ones.

While alloys can improve the accuracy of model inputs, there are aspects of the model itself which are substantial approximations. In reality, extrusion speed and temperature

significantly affect the microstructure, and hence strength, of a material. To maintain constant microstructure and properties throughout, modern extrusion systems use computers to adjust temperature and speed in real-time. Too high temperatures can result in surface cracks, while extruding at too low temperatures results in increased pressure and reduced tool life [14]. Hence, cycle times and material strength can vary in ways the model does not account for.

Broadly speaking, the practical usefulness of the three metrics of strength used in the model – Euler buckling load and deflection under center and end load – varies depending on application. For intense axial loading, in a collision for example, plastic deformation and fracture is almost always the dominant mode of failure. Indeed, the geometries considered in this analysis have too low a slenderness ratio for the Euler formula to be practical for ordinary design – such columns would fail by a combination of bending and plastic deformation [16]. Further, all three metrics of strength are highly dependent on end conditions which may or may not be realistic. While theoretically ‘fixed’, end conditions can quickly change in a high impact collision as joints fail. Further, the idealized point center and end loads are likely to be somewhat distributed, not to mention the fact that the unsupported end of a cantilever is more a theoretical construction than practical reality. So while interpreting results in *absolute* terms is a mistake, one can still gain insight into strength differences between materials on a *relative* basis. That is where this model provides true value.

Finally, it’s important to note that the case study kept length constant at 1m for simplicity, but in reality length can vary substantially. In terms of axial loading, doubling length quarters the critical load, while for deflection, longer members deflect more. In general, shorter member are stronger, so while kept constant in this study for simplicity, length modifications may be another avenue of optimization in some applications. Of course, the shorter the beam, the less practically useful the strength metrics used in the model will be.

6 Conclusions and Future Work

In this thesis, we have developed a framework within a Process Based Cost Model of extrusion for understanding the strength characteristics of extruded beams in different loading scenarios, and further, how these characteristics, via processing parameters and geometry, translate into cost and weight savings. Consisting of a suite of mechanics and cost functions developed in Visual Basic, this framework dynamically interfaces with the cost model - taking as inputs material and part properties, computing dimensions subject to strength constraints, inputting these values into the model, and returning cost output.

Through a case study, we have demonstrated this capability on a simple system which approximates a number of real life components in automotive applications. Specifically, through systematic variation of parameters and loading scenarios, we have utilized the 'Strength Analysis' tab to understand the conditions in which it is cost effective for a manufacturer to switch from Al to Mg in order to take advantage of its light weighting potential. This included indentifying the key drivers of cost as well as defining how variations of process, strength, and geometric parameters affect the desirability of Mg over Al.

Future work includes performing similar analysis on systems of different diameters, thicknesses, and lengths, as well improving the accuracy of various model inputs. Further, while currently constrained to three loading scenarios with either Mg or Al, new loading scenarios and materials could be incorporated by making minor changes to the code. Specifically, Mg-Al alloys would be interesting to study as well as redefining the model's metric of axial strength. Indeed, pure Euler buckling is not a realistic mode of failure for structural members of slenderness ratio less than 50, such as those considered in this analysis. Great improvement can be made by replacing this strength metric with empirical formulae designed specifically for modeling beams of intermediate slenderness ratio [14]. Unfortunately, this realization was made far too late into the semester to be corrected in this thesis, and further, while referenced online, these empirical methods could not be located. Nevertheless, the methods described herein represent a substantial improvement in the features and practical usefulness of the MSL Extrusion cost model.

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8 APPENDIX

Mechanics Functions

```
Public Function BucklingCalc(Material As String, DimensionToVary As String,  
DimensionGiven As Double, CriticalLoad As Double) As Double
```

```
Dim TestLength As Double  
Dim Young As Double  
Dim TestRad As Double  
Dim TestRadInner As Double  
Dim TestWall As Double  
Dim a As Double  
Dim b As Double  
Dim c As Double  
Dim d As Double  
Dim p As Double  
Dim q As Double  
Dim r As Double  
Dim Test As Double
```

```
TestLength = Application.Range("TEST_LEN")
```

```
If Material = "Al" Then  
    Young = 70 * 10 ^ 9  
ElseIf Material = "Mg" Then  
    Young = 45 * 10 ^ 9  
Else: BucklingCalc = CVErr(xlErrNA)  
End If
```

```
Select Case DimensionToVary
```

```
Case "Thickness"
```

```
    TestRad = DimensionGiven / 2  
    TestRadInner = (((TestRad / 1000) ^ 4 - ((CriticalLoad * 10 ^ 6 * TestLength ^  
2) / ((Application.WorksheetFunction.Pi() ^ 3) * Young))) ^ 0.25) * 1000  
    BucklingCalc = TestRad - TestRadInner
```

```
Case "Diameter"
```

'Formula for solving cubic polynomial at <http://www.math.vanderbilt.edu/~schectex/courses/cubic/>

```
    TestWall = DimensionGiven / 1000  
  
    a = 4 * TestWall  
    b = -6 * TestWall ^ 2  
    c = 4 * TestWall ^ 3  
    d = -TestWall ^ 4 - ((CriticalLoad * 10 ^ 6 * TestLength ^ 2) /  
((Application.WorksheetFunction.Pi() ^ 3) * Young))  
  
    p = -b / (3 * a)  
    q = p ^ 3 + (b * c - 3 * a * d) / (6 * a ^ 2)
```

```

r = c / (3 * a)

Test = (q - (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2))

If Test < 0 Then
    TestRad = ((q + (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) - (Math.Abs(q -
(q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2))) ^ (1 / 3) + p) * 1000
    Else: TestRad = ((q + (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) + (q - (q ^ 2
+ (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) + p) * 1000
End If

BucklingCalc = 2 * TestRad

End Select

End Function

```

```

Public Function DeflectionCalc(LoadType As String, Material As String,
DimensionToVary As String, DimensionGiven As Double, DeflectionPercent As
Double, Load As Double) As Double

```

```

Dim TestLength As Double
Dim Young As Double
Dim TestRad As Double
Dim TestRadInner As Double
Dim TestWall As Double
Dim Deflection As Double
Dim a As Double
Dim b As Double
Dim c As Double
Dim d As Double
Dim p As Double
Dim q As Double
Dim r As Double
Dim Test As Double

```

```

TestLength = Application.Range("TEST_LEN")
Deflection = TestLength * DeflectionPercent

```

```

If Material = "Al" Then
    Young = 70 * 10 ^ 9
ElseIf Material = "Mg" Then
    Young = 45 * 10 ^ 9
Else: DeflectionCalc = CVErr(xlErrNA)
End If

```

```

Select Case DimensionToVary

```

```

Case "Thickness"

```

```

    TestRad = DimensionGiven / 2

```

```

If LoadingType = "End" Then
    TestRadInner = (((TestRad / 1000) ^ 4 - ((4 * Load * 10 ^ 3 * TestLength ^ 3) / (3 * Application.WorksheetFunction.Pi() * Young * Deflection)))) ^ 0.25) * 1000
    DeflectionCalc = TestRad - TestRadInner
ElseIf LoadingType = "Center" Then
    TestRadInner = (((TestRad / 1000) ^ 4 - ((Load * 10 ^ 3 * TestLength ^ 3) / (3 * Application.WorksheetFunction.Pi() * Young * Deflection)))) ^ 0.25) * 1000
    DeflectionCalc = TestRad - TestRadInner
Else: DeflectionCalc = CVErr(xlErrNA)
End If

Case "Diameter"

'Formula for solving cubic polynomial at http://www.math.vanderbilt.edu/~schectex/courses/cubic/
TestWall = DimensionGiven / 1000

a = 4 * TestWall
b = -6 * TestWall ^ 2
c = 4 * TestWall ^ 3

If LoadingType = "End" Then
    d = -TestWall ^ 4 - ((4 * Load * 10 ^ 3 * TestLength ^ 3) / (3 * Application.WorksheetFunction.Pi() * Young * Deflection))
ElseIf LoadingType = "Center" Then
    d = -TestWall ^ 4 - ((Load * 10 ^ 3 * TestLength ^ 3) / (3 * Application.WorksheetFunction.Pi() * Young * Deflection))
Else: DeflectionCalc = CVErr(xlErrNA)
End If

p = -b / (3 * a)
q = p ^ 3 + (b * c - 3 * a * d) / (6 * a ^ 2)
r = c / (3 * a)

Test = (q - (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2))

If Test < 0 Then
    TestRad = ((q + (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) - (Math.Abs(q - (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2))) ^ (1 / 3) + p) * 1000
Else: TestRad = ((q + (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) + (q - (q ^ 2 + (r - p ^ 2) ^ 3) ^ (1 / 2)) ^ (1 / 3) + p) * 1000
End If

DeflectionCalc = 2 * TestRad

End Select

End Function

```

```

Public Function Deflection(LoadType As String, Material As String, Length As Double, Thickness As Double, Diameter As Double, Load As Double) As Double

```

```

    Dim Moment As Double

```

```

If Material = "Al" Then
    Young = 70 * 10 ^ 9
ElseIf Material = "Mg" Then
    Young = 45 * 10 ^ 9
Else: Deflection = CVErr(xlErrNA)
End If

```

```

Moment = (Application.WorksheetFunction.Pi / 4) * ((Diameter / 2 / 1000) ^ 4 -
((Diameter / 2 - Thickness) / 1000) ^ 4)

```

```

If LoadingType = "End" Then
    Deflection = (Load * 10 ^ 3 * Length ^ 3) / (3 * Young * Moment)
ElseIf LoadingType = "Center" Then
    Deflection = (Load * 10 ^ 3 * Length ^ 3) / (12 * Young * Moment)
Else: Deflection = CVErr(xlErrNA)
End If

```

```

End Function

```

```

Public Function Load(LoadType As String, Material As String, Thickness As Double,
Diameter As Double, Length As Double, DeflectionPercent As Double) As Double

```

```

    Dim Moment As Double
    Dim Deflection As Double

```

```

    Deflection = Length * DeflectionPercent

```

```

If Material = "Al" Then
    Young = 70 * 10 ^ 9
ElseIf Material = "Mg" Then
    Young = 45 * 10 ^ 9
Else: Load = CVErr(xlErrNA)
End If

```

```

Moment = (Application.WorksheetFunction.Pi / 4) * ((Diameter / 2 / 1000) ^ 4 -
((Diameter / 2 - Thickness) / 1000) ^ 4)

```

```

If LoadingType = "End" Then
    Load = ((3 * Deflection * Young * Moment) / (Length ^ 3)) / 10 ^ 3
ElseIf LoadingType = "Center" Then
    Load = ((12 * Deflection * Young * Moment) / (Length ^ 3)) / 10 ^ 3
Else: Deflection = CVErr(xlErrNA)
End If

```

```

End Function

```

```

Public Function Weight(Material As String, Thickness As Double, Diameter As Double,
Length As Double) As Double

```


Dim Density As Double

```
If Material = "Al" Then
Density = 2700
Else: Density = 1800
End If
```

```
Weight = Density * Application.WorksheetFunction.Pi() * (((Diameter / 2) ^ 2 -
(Diameter / 2 - Thickness) ^ 2) / (1000 ^ 2)) * Length
```

End Function

Cost Functions

Public Function CostCalcBuckling(Material As String, DimensionToVary As String,
DimensionGiven As Double, CriticalLoad As Double, CostUnit As String) As Double

Dim Diameter As Double
Dim Thickness As Double

Select Case DimensionToVary

Case "Thickness"

```
Diameter = DimensionGiven
Thickness = BucklingCalc(Material, DimensionToVary, Diameter, CriticalLoad)
Application.Range("TEST_DIA").Value = Diameter
Application.Range("TEST_WALL").Value = Thickness
```

If CostUnit = "kg" Then

```
CostCalcBuckling = Application.Range("TOTAL_FABRICATION_COST_UNIT") /
Application.Range("TEST_WT")
ElseIf CostUnit = "unit" Then
CostCalcBuckling = Application.Range("TOTAL_FABRICATION_COST_UNIT")
Else: CostCalcBuckling = CVErr(xlErrNA)
End If
```

Case "Diameter"

```
Thickness = DimensionGiven
Diameter = BucklingCalc(Material, DimensionToVary, Thickness, CriticalLoad)
Application.Range("TEST_DIA").Value = Diameter
Application.Range("TEST_WALL").Value = Thickness
```

If CostUnit = "kg" Then

```
CostCalcBuckling = Application.Range("TOTAL_FABRICATION_COST_UNIT") /
Application.Range("TEST_WT")
Else: CostCalcBuckling = Application.Range("TOTAL_FABRICATION_COST_UNIT")
End If
```

End Select

End Function

```
Public Function CostCalcDeflection(LoadType As String, Material As String,  
DimensionToVary As String, DimensionGiven As Double, DeflectionPercent As  
Double, Load As Double, CostUnit As String) As Double
```

```
    Dim Diameter As Double  
    Dim Thickness As Double
```

```
    Select Case DimensionToVary
```

```
        Case "Thickness"
```

```
            Diameter = DimensionGiven  
            Thickness = DeflectionCalc(LoadType, Material, DimensionToVary, Diameter,  
DeflectionPercent, Load)  
            Application.Range("TEST_DIA").Value = Diameter  
            Application.Range("TEST_WALL").Value = Thickness  
            Application.Range("FORCE").Value = Load
```

```
            If CostUnit = "kg" Then  
                CostCalcDeflection = Application.Range("TOTAL_FABRICATION_COST_UNIT") /  
Application.Range("TEST_WT")  
            ElseIf CostUnit = "unit" Then  
                CostCalcDeflection = Application.Range("TOTAL_FABRICATION_COST_UNIT")  
            Else: CostCalcDeflection = CVErr(xlErrNA)  
            End If
```

```
        Case "Diameter"
```

```
            Thickness = DimensionGiven  
            Diameter = DeflectionCalc(LoadType, Material, DimensionToVary, Thickness,  
DeflectionPercent, Load)  
            Application.Range("TEST_DIA").Value = Diameter  
            Application.Range("TEST_WALL").Value = Thickness  
            Application.Range("FORCE").Value = Load
```

```
            If CostUnit = "kg" Then  
                CostCalcDeflection = Application.Range("TOTAL_FABRICATION_COST_UNIT") /  
Application.Range("TEST_WT")  
            Else: CostCalcDeflection = Application.Range("TOTAL_FABRICATION_COST_UNIT")  
            End If
```

```
    End Select
```

```
End Function
```

Cost Calculation Subroutines

```
Sub PerformBucklingCostCalculations()
```

```
Dim CostCell As String  
Dim ForceCell As String  
Dim DataPoints As Integer  
Dim Index As Integer  
Dim ComputedCost As Double  
Dim Material As String  
Dim ComputedDimension As String  
Dim GivenDimension As Double  
Dim CostUnit As String
```

```
CostCell = Application.Range("TOP_COST_CELL").Value  
ForceCell = Application.Range("TOP_FORCE_CELL").Value  
DataPoints = Application.Range("DATA_POINTS").Value  
CostUnit = Application.Range("COST_UNIT").Value
```

```
Material = Application.Range("TEST_MAT_NAME").Value  
ComputedDimension = Application.Range("COMPUTED_DIM").Value  
GivenDimension = Application.Range("GIVEN_DIM").Value
```

```
Index = 0
```

```
Do While Index < DataPoints
```

```
ComputedCost = CostCalcBuckling(Material, ComputedDimension, GivenDimension,  
Application.Range(ForceCell).Offset(Index, 0), CostUnit)  
Application.Range(CostCell).Offset(Index, 0).Value = ComputedCost  
Application.Range(CostCell).Offset(Index, 0).Font.ColorIndex = 5  
Index = Index + 1
```

```
Loop
```

```
End Sub
```

```
Sub PerformDeflectionCostCalculations()
```

```
Dim CostCell As String  
Dim DeflectionOrLoadCell As String  
Dim DataPoints As Integer  
Dim Index As Integer  
Dim ComputedCost As Double  
Dim Material As String  
Dim ComputedDimension As String  
Dim GivenDimension As Double  
Dim CostUnit As String  
Dim LoadingType As String  
Dim Load As Double  
Dim Deflection As Double
```

```

CostCell = Application.Range("TOP_COST_CELL2").Value
DataPoints = Application.Range("DATA_POINTS2").Value
CostUnit = Application.Range("COST_UNIT2").Value
LoadingType = Application.Range("LOADING_TYPE").Value
Material = Application.Range("TEST_MAT_NAME2").Value
ComputedDimension = Application.Range("COMPUTED_DIM2").Value
GivenDimension = Application.Range("GIVEN_DIM2").Value

DeflectionOrLoadCell =
Application.Range("TOP_DEFLECTION_OR_LOAD_CELL").Value
Index = 0

If Application.Range("FIXED_PARAMETER").Value = "Load" Then
    Load = Application.Range("FIXED_PARAMETER_VALUE").Value

    Do While Index < DataPoints

        ComputedCost = CostCalcDeflection(Load, Material, ComputedDimension,
        GivenDimension, Application.Range(DeflectionOrLoadCell).Offset(Index, 0), Load,
        CostUnit)
        Application.Range(CostCell).Offset(Index, 0).Value = ComputedCost
        Application.Range(CostCell).Offset(Index, 0).Font.ColorIndex = 5
        Index = Index + 1

    Loop

ElseIf Application.Range("FIXED_PARAMETER").Value = "Deflection" Then
    Deflection = Application.Range("FIXED_PARAMETER_VALUE").Value

    Do While Index < DataPoints

        ComputedCost = CostCalcDeflection(Load, Material, ComputedDimension,
        GivenDimension, Deflection, Application.Range(DeflectionOrLoadCell).Offset(Index,
        0), CostUnit)
        Application.Range(CostCell).Offset(Index, 0).Value = ComputedCost
        Application.Range(CostCell).Offset(Index, 0).Font.ColorIndex = 5
        Index = Index + 1

    Loop

Else:
End If

End Sub

```

Press Size Determination

```

Public Function PressSize() As Double

    Dim Part As Integer
    Dim Mat As Integer

```

```

Dim Index As Integer

Dim Force As Double
Dim Pressure As Double
Dim ShapeFactor As Double
Dim ReductionRatio As Double
Dim BilletLength As Double
Dim BilletDiameter As Double
Dim Circum As Double
Dim CircumEquiv As Double
Dim BilletArea As Double
Dim FinalArea As Double

Dim ExtrusionConstant As Double

Dim Compatible As Boolean

Compatible = False
Part = Application.Range("PART")
Mat = Application.Range("MAT")
Index = 3

PressSize = Application.Range("PRESS_TABLE_2").Cells(1, Index)

Do While Compatible = False

    BilletDiameter = Application.WorksheetFunction.HLookup(PressSize,
Application.Range("PRESS_TABLE_2"), 2)

    BilletArea = Application.WorksheetFunction.Pi * (BilletDiameter / 2) ^ 2
    FinalArea = Application.WorksheetFunction.VLookup(Part,
Application.Range("PART_DAT"), 12)

    ReductionRatio = BilletArea / FinalArea

    Circum = Application.WorksheetFunction.VLookup(Part,
Application.Range("PART_DAT"), 10)
    CircumEquiv = Application.WorksheetFunction.VLookup(Part,
Application.Range("PART_DAT"), 11)

    ShapeFactor = 0.98 + 0.02 * (Circum / CircumEquiv) ^ 2.25
    ExtrusionConstant = Application.WorksheetFunction.VLookup(Mat,
Application.Range("MAT_DAT"), 10)

    Pressure = ShapeFactor * ExtrusionConstant * Log(ReductionRatio)
    Force = Pressure * BilletArea

    If Force < PressSize Then
        Compatible = True
        Exit Do

    Else:
        Index = Index + 1

```

```
        PressSize = Application.Range("PRESS_TABLE_2").Cells(1, Index)
    End If

    Loop

End Function
```